

Sufficient Statistics for Frictional Wage Dispersion and Growth *

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Abstract

This paper develops a sufficient statistics approach for estimating the role of search frictions in wage dispersion and lifecycle wage growth. We show how the wage dynamics of displaced workers are directly informative of both for a large class of search models. Specifically, the correlation between pre- and post-displacement wages is informative of frictional wage dispersion. Furthermore, the fraction of displaced workers who suffer a wage loss is informative of frictional wage growth and job-to-job mobility, independent of the job-offer distribution and other labor-market parameters. Applying our methodology to US data, we find that search frictions account for less than 20 percent of wage dispersion. In addition, we estimate that between 40 to 80 percent of workers experience no frictional wage growth during an employment spell. Our approach allows us to estimate how frictions change over time. We find that frictional wage dispersion has declined substantially since 1980 and that frictional wage growth, while low, is more important towards the end of expansionary periods. We finish by estimating two versions of a random search model to show how at least two different mechanisms—involuntary job transitions or compensating differentials—can reconcile our results with the job-to-job mobility seen in the data. Regardless of the mechanism, the estimated models show that frictional wage growth accounts for about 15 percent of lifecycle wage growth.

Keywords: Search Models, Wage Dispersion, Wage Growth, Sufficient Statistics, Displacement
JEL codes: E24, J31, J64

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1 Introduction

Search models are considered foundational models in the macro-economist’s toolbox (Blanchard 2018). These models have been used to help economists understand, among other things, wage inequality (*e.g.* Mortensen 2005), wage growth (*e.g.* Burdett and Mortensen 1998), the allocation of workers to firms (*e.g.* Shimer and Smith 2000), and the Philips Curve (*e.g.* Moscarini and Postel-Vinay 2019). Two core elements of search models are important for understanding the role of search frictions in labor markets. First, how relatively disperse are the job opportunities that workers face? Second, how easily do workers find better job opportunities during an employment spell? In other words, how important are the frictional components of wage dispersion and wage growth. These elements are complementary for many questions that search models seek to answer, but are not easy to identify in typical labor market datasets.

In the spirit of the sufficient statistics literature, we propose two simple statistics that are informative of frictional wage dispersion and frictional wage growth in search models.¹ The statistics have the advantage that they only require information on the wage dynamics of displaced workers and can be estimated from cross-sectional or short panel datasets. The first main identifying assumption is that conditional on worker type the post-displacement wage is statistically independent of the pre-displacement wage. The second main identifying assumption is that we can construct a sample of workers that have been exogenously displaced.² Inference using these statistics is independent of wage offer distributions and other labor market parameters. While the empirical wage losses of displaced workers has been studied extensively and has also been used in the estimation of search models, the main contribution of this paper is different. First, we show how these statistics are directly informative of frictional wage dispersion, frictional wage growth, and job-to-job mobility parameters under minimal assumptions. Second, we propose an empirical strategy and tests that lend credibility to this approach more generally (*i.e.* satisfy the assumptions), whether using these statistics directly or as a part of a structural estimation strategy.

The proposed *wage-dispersion statistic* is the correlation between the pre- and post-displacement wage. We show that the wage-dispersion statistic describes the fraction of the variance of wages accounted for by the between-worker variation in the cross-sectional wage distribution. The remaining variation can be considered an upper bound for the contribution of frictional wage dispersion as it could include variation due to firm-specific human capital, measurement error, compensating wage differentials, *etc.* The wage-dispersion statistic is informative of frictional wage dispersion across most search models in the literature.

The proposed *wage-growth statistic* is the fraction of displaced workers earning lower wages after displacement. This direct measure of frictional wage growth is related to the average wage loss commonly used in the literature. The fraction is a more direct measure of how far workers have

¹See Chetty (2009) for a survey of the sufficient statistics literature.

²In other words, displacement is independent of the worker’s pre-displacement wage, conditional on worker type. We show empirical evidence supporting this assumption in Section 3.2.

climbed the wage ladder as it is not sensitive to details of the offer distribution, which the average wage loss is. Using wage dynamics to make inference on job-to-job mobility will depend more on the details of the model, but we can still make inference for a large class of models. In order to make inference about job-to-job mobility, we need to additionally assume, first, that jobs are rank preserving;³ second, that workers choose the job with the higher wage; and, third, workers draw from the same offer distribution independent of employment status. One advantage of our approach is that it does not require job arrival rates and wage distributions to be time-invariant; only that the rankings of jobs are time-invariant.⁴ Without making additional assumptions about the offer arrival technology (*e.g.* a Poisson process), we show that the wage-growth statistic can be used to place bounds on the number of job offers workers receive during an employment spell. In order to compare our results to a common measure of job-to-job mobility in the literature, we show how the wage-growth statistic can be directly related to the ratio of the Poisson rate of job offers while employed to the Poisson rate of job separations, often represented in the literature by $\kappa = \lambda^e / \delta$. The ratio κ is a key parameter in search models as it describes labor market competition and wage growth.⁵ As before, wage distributions and other labor market parameters—such as κ —are allowed to be worker-specific and we do not need to make steady-state assumptions. Two important classes of models, sequential-auction models and compensating-differentials models, may violate the assumption that workers always choose the job with the higher wage.⁶ We show that under certain conditions, the inferred κ from the wage-growth statistic is an upper bound for the κ in a sequential-auction model. In Section 4, we show that the wage-growth statistic is, in practice, informative about frictional wage growth in compensating differential models.

We estimate the statistics using two U.S. datasets: the Displaced Worker Supplement to the Current Population Survey (CPS-DWS) and the Survey of Income and Program Participation (SIPP). The CPS-DWS is a cross-sectional dataset that identifies workers who were displaced by a plant closure and measures weekly wages. The SIPP measures monthly wages and identifies workers who were displaced by permanent layoffs, slack work conditions, or firm bankruptcy. We view the datasets as complementary, since they survey displaced workers differently (retrospective vs. panel) and record post-displacement wages at different points in time. In both datasets, we select a sample of full-time private sector workers who were displaced and found full-time private sector jobs after displacement. We test and do not find any difference between the pre-displacement wages in our sample and the wages of cross-sectional workers. We also test and do not find evidence that workers with higher pre-displacement wages wait longer to accept a job. We estimate both statistics, with and without measurement error corrections. We find a correlation between pre-

³Rank preserving jobs means that if a worker prefers job A over job B at a given time, then the worker will always prefer job A over job B.

⁴The time-invariant ranking assumption is consistent with the search literature studying business cycles and human capital accumulation. See *e.g.* Hagedorn and Manovskii (2013) and Bagger, Fontaine, Postel-Vinay, and Robin (2014).

⁵See *e.g.* Burdett and Mortensen (1998).

⁶See *e.g.* Cahuc, Postel-Vinay, and Robin (2006) for a sequential auction model and Taber and Vejlín (2020) for a compensating differential model.

displacement and post-displacement wages (the wage-dispersion statistic) of 0.76 and 0.72 in the SIPP and CPS-DWS, respectively. After correcting for measurement error, we infer that frictional wage dispersion accounts for less than 20% of the variance of wages. In addition, we find that about 58% of displaced workers earn lower wages after displacement in both the SIPP and CPS-DWS (the wage-growth statistic).⁷⁸ Turning to job-to-job mobility, the estimated wage-growth statistics imply that 40% – 80% percent of workers receive zero job offers during an employment spell (*i.e.* experience no frictional wage growth). Assuming a Poisson offer arrival technology, the wage-growth statistic implies values of κ of 0.87 and 0.73, for the SIPP and CPS-DWS respectively. In other words, workers are more likely to receive a job separation shock than a job offer.

We also estimate the statistics by education groups and across time. While the wage-dispersion statistic does not differ much by education, the wage-growth statistic is substantially lower for college graduates. The estimates for college graduates imply that at least 60 percent of college graduates receive zero job offers during an employment spell and are at least twice as likely to receive a job separation shock than a job offer. Finally, we find that frictional wage dispersion has been declined by about half relative to total wage dispersion over the last 30 years.

Our estimates of low relative offer rates may seem to be at odds with the large job-to-job flows observed in the US labor market.⁹ We consider two mechanisms that can reconcile our estimated wage-growth statistic and observed job-to-job transition rates: involuntary job offers and compensating differentials.¹⁰ We show how an involuntary job offer "resets" the frictional wage growth process and we relate our wage-growth statistic to these classes of models with minor modifications. A compensating differential model is one where jobs offer a non-pecuniary benefit and workers may change jobs in order to get a higher non-pecuniary benefit even if it means a lower wage. Both extensions can accommodate large job-to-job flows with low frictional wage growth. Both mechanisms highlight that one should be careful inferring frictional wage growth from job-to-job transition rates alone.

We proceed by setting up and estimating two versions of a random search model with two-sided heterogeneity and general human capital. The first version includes involuntary job offers and the second version includes compensating differentials. The goals of structurally estimating the two models are three-fold. First, we show that either of the two extensions can reconcile our results with observed transition rates. Second, we show that inferring job offer rates from observed transition rates is highly model-dependent. In other words, a large set of job offer rates is consistent with the same observed job transition rate and frictional wage growth rate depending on modelling

⁷Our estimates for the average wage loss from displacement are similar to the literature. We find average wage losses of about 7% and 9% for workers with at least one and three years of tenure, respectively. See Table 7 for more information.

⁸This is not the first paper to find that a significant fraction of workers earn higher wages after displacement. See *e.g.* Fallick, Haltiwanger, McEntarfer, and Staiger (2019), Hyatt and McEntarfer (2012), and Farber (2017).

⁹Using job-to-unemployment and job-to-job flows implies a κ that is 4.5 times higher than what we estimate from the wage-growth statistic. See footnote 53.

¹⁰An involuntary job offer is one where the outside option for the worker is not staying with the current employer, but to go to unemployment. One can think of this as capturing advance notice or workers having to find a new job for personal reasons (*e.g.* spouse needs to move).

assumptions. Finally, we use the models to quantify the fraction of total wage growth over the life-cycle explained by frictional wage growth.

Both estimated models match the frictional wage statistics and the observed transition rates in the data. The inferred job offer rates when employed are quite different though. The compensating-differentials model estimates a job offer rate that is three times higher than the model with involuntary job offers. This is expected as workers accept a higher fraction of job offers if their frictional wage growth path is occasionally reset by involuntary job offers. Since workers accept a higher fraction of job offers, a lower job offer rate is needed to match the observed job-to-job transition rate. Regardless of the mechanism, the estimated models show that frictional wage growth accounts for about 15 percent of lifecycle wage growth and frictional wage dispersion accounts for about 16-17 percent of wage dispersion over the first 25 years of the lifecycle. Finally, we estimate models with correlated productivity and non-pecuniary benefits, differences in job offer distributions between employed and unemployed, and a joint model with both non-pecuniary benefits and involuntary job offers. All models give similar results regarding the importance of frictions.

This paper relates to a large empirical literature studying frictional wage dispersion and frictional wage growth. The papers most related to ours study these topics without estimating a full structural model (*e.g.* without assuming functional forms for the wage offer distribution). One of the most well-known examples is [Hornstein, Krusell, and Violante \(2011\)](#), which use unemployment durations (*i.e.* job offer rates) to infer the importance of frictional wage dispersion. On one hand, they find that statistics of labor-market turnover rates imply very little frictional wage dispersion in a basic search model without on-the-job search. On the other hand, they find that models with on-the-job search can be consistent with both the turnover rates and important contributions from frictional dispersion. [Alvarez, Borovickova, and Shimer \(2014\)](#) use an estimator related to our wage-dispersion statistic to study heterogeneity in unemployment duration and wages using data on workers who experience two different unemployment spells in administrative data from Austria. Lastly, [Barlevy \(2008\)](#) uses detailed job history data to identify κ , which allows him to non-parametrically estimate the offer distribution in the Burdett-Mortensen model.¹¹ Finally, our paper is also connected to the empirical literature pioneered by [Abowd, Kramarz, and Margolis \(1999\)](#) (AKM) that estimate fixed effect models with firm and worker fixed effects. The estimated models using the AKM framework are typically used to answer questions about the drivers of wage dispersion through different decompositions of wage variation. Our approach has several advantages compared to the AKM approach, since we do not need to worry about incidental parameters bias for the fixed effects, endogenous mobility, or assuming fixed types over long periods. Thus, the ability of our method to deal with short panels is an important improvement over AKM. Our methodology complements the literature more generally in using wage statistics instead of labor-market turnover and it can be estimated using publicly-available labor-market datasets compared to *e.g.* AKM, which require detailed matched employer-employee datasets.

¹¹Another recent example is [Gottfries and Teulings \(2021\)](#), who use variation in job-finding rates and time since last lay-off to separate frictional wage growth from human capital wage growth.

This paper relates to a parallel literature estimating structural search models to understand the role of frictional wage dispersion.¹² Some of the earliest contributions to this literature are [Wolpin \(1992\)](#), [Van den Berg and Ridder \(1998\)](#) and [Bontemps, Robin, and van den Berg \(1999\)](#). [Wolpin \(1992\)](#) investigates the first five years post-schooling for blacks and whites with a high school degree and finds that the wage offer distribution is more compressed for blacks, while they have higher off and on the job offer rates. The latter two papers extend the wage posting model of [Burdett and Mortensen \(1998\)](#) and add firm heterogeneity in productivity. [Van den Berg and Ridder \(1998\)](#) assumes that worker heterogeneity comes from observable differences, while [Bontemps, Robin, and van den Berg \(1999\)](#) assumes that worker heterogeneity arise from differences in worker’s opportunity costs of employment. Both studies find that search frictions play an important role. Specifically, [Van den Berg and Ridder \(1998\)](#) finds that search frictions explain around 20 percent of the variance of wage offers. [Postel-Vinay and Robin \(2002\)](#) were the first to estimate a sequential-auction model with unobserved worker and firm heterogeneity. They estimate that search frictions explain around 40-60 percent of cross-sectional wage variation. More recently, [Taber and Vejlin \(2020\)](#) extended the model of [Postel-Vinay and Robin \(2002\)](#) to include human capital, compensating differentials, and multidimensional pre-market skills. They find that search frictions play a minor role in explaining cross-sectional wage variation. [Tjaden and Wellschmied \(2014\)](#) includes involuntary reallocation shocks in a search model and estimates that around 15 percent of cross-sectional wage dispersion is due to search frictions.

There has also emerged a literature structurally estimating the role of frictions on wage growth. This literature goes back to [Topel and Ward \(1992\)](#). More recently, [Bagger, Fontaine, Postel-Vinay, and Robin \(2014\)](#) extended the model of [Cahuc, Postel-Vinay, and Robin \(2006\)](#) to encompass human capital and decomposed wage growth over the life-cycle into search induced growth and human capital wage growth. They find that the wage-experience profile is explained by both search frictions and human capital, but with search frictions typically playing the main role. However, they estimate their model on Danish data and not US data as we do. [Yamaguchi \(2010\)](#) also estimate a model with both human capital and search frictions, but on US data. He finds that search frictions contribute to wage growth significantly in the first 5 years, but that human capital is the main driver of wage growth after that. This is very similar to our findings.¹³

The paper is divided into three parts. In Section 2, we show how our two wage statistics are informative of fundamental parameters in a large class of search models. In Section 3, we estimate these statistics using US data. In Section 4, we estimate two models using our statistics. In Section 5, we conclude the paper.

¹²There is also a literature using structural search models to understand the earnings consequences of job loss, see e.g. [Jarosch \(2021\)](#), [Krolikowski \(2017\)](#), [Jung and Kuhn \(2018\)](#). These papers typically look at all unemployment spells and often focus on high-tenure workers thus generating larger earnings losses than what we find.

¹³Other papers in this literature include [Burdett, Carrillo-Tudela, and Coles \(2011\)](#), [Bowlus and Liu \(2013\)](#), and [Menzio, Telyukova, and Visschers \(2016\)](#).

2 Statistics for Frictional Wage Dispersion and Frictional Wage Growth

In this section, we show how our statistics relate to frictional wage dispersion and frictional wage growth in search models. Before discussing each statistic, we describe the basic environment and the main identifying assumptions used for both statistics.

2.1 Economic Environment

Consider an economy populated with heterogeneous workers, where worker heterogeneity is described by discrete types, $x \in \mathcal{X}$. A worker's type may evolve over time (*e.g.* due to human capital accumulation). Let the worker type of worker i at calendar time t be denoted by x_{it} . The state of the economy might change over time due to, for example, economic conditions (*e.g.* business cycle dynamics). Let s_t denote the state of the economy at time t . s_t could be specific to the local labor market where the worker lives. Unemployed workers of type x draw job offers from a well-behaved job offer distribution function $F_{xs}(w)$. The cumulative distribution function, $F_{xs}(w)$, describes wage offers that are above a worker's reservation wage. Let a different well-behaved distribution function, $G_{xs}(w)$, describe the wages of employed workers of type x when the economy is in state s in the cross-section.

Frictional wage dispersion and frictional wage growth are identified using the wages of displaced workers who experience an employment-unemployment-employment (JUU) transition. Consider workers of type x who are displaced from their job when the state of the economy is s_t . Let w^{pre} denote a worker's pre-displacement wage and w^{post} denote the post-displacement wage. Three main assumptions are needed for identification.¹⁴

A1.1 : Independence of post-displacement wages: $(w^{post} \perp\!\!\!\perp w^{pre}) | x, s$

The first assumption is that, conditional on worker type x and the state of the economy s , the post-displacement wage is independent of the pre-displacement wage for each worker. In other words, Assumption A1.1 states that the offer distribution is independent of the pre-displacement wage conditional on the worker type ($F_{xs}(w|w^{pre}) = F_{xs}(w)$). While the independence assumption holds in most search models, it may be violated if the reservation wage depends on a worker's wage history.¹⁵ Various economic mechanisms (*e.g.* savings, habit formation, expectation of recall, etc) may induce workers with higher pre-displacement wages to have higher reservation wages and to search longer on average. We investigate this empirically in Section 3.2 and do not find evidence of a relationship between the pre-displacement wage and unemployment duration.

A1.2 : Exogeneity of displacement: $\Pr(displacement|w^{pre}, x, s) = \Pr(displacement|x, s)$

¹⁴Inference on job-to-job mobility requires additional assumptions as explained in Section 2.3.

¹⁵Assumption A1.1 would also be violated if workers could recall previous job offers as in Carrillo-Tudela and Smith (2017).

The second assumption is that displacement by plant closure (or mass layoff) is independent of the wage conditional on the worker type and the state of the economy. While considering plant closures as exogenous layoffs is common in the empirical displacement literature, assuming that these displacements are unrelated to pre-displacement wages is less common. Assumption A1.2 implies that the lower wages found at closing plants is due to the worker type composition. The intuition is that the likelihood of a plant closure will depend on the technology and composition of the workers employed. A plant using obsolete technology is more likely to close when it receives a negative shock (*e.g.* consumer preferences shift or a new competing product enters the market). The portion of workers using obsolete or non-transferable skills will have a hard time finding a new job after displacement and also will have had lower wages since there is little demand for those skills. Workers with transferable skills find new jobs and have pre-displacement wages similar to the cross-section due to competition from other firms. In Section 3.2, we perform a number of empirical tests showing that, conditional on education, the pre-displacement wage distribution of JJJ workers exogenously displaced (*e.g.* by a plant closure) and the wage distribution of employed workers in the cross-section are statistically indistinguishable. The fraction of workers who do not find a job within 12 months have pre-displacement wages that are lower than the cross-section.¹⁶

A1.3 : Worker types and economic conditions do not change over an unemployment spell: $x_{it}(t = t^{pre}) = x_{it}(t = t^{post})$ and $s_t(t = t^{pre}) = s_t(t = t^{post})$

The last assumption is that pre-displacement and post-displacement wages are measured close enough in time that the worker type and wage distributions, which are functions of x and s , do not change between displacement and finding a new job. Assumption A1.3 is needed as our empirical strategy relies on making comparisons between the pre-displacement and post-displacement wages. Our sample is restricted to workers who find a job within a year for this reason.

2.2 A Statistic for Frictional Wage Dispersion

The wage-dispersion statistic is the correlation between the pre- and post-displacement wage. The correlation measures the persistence of wages across JJJ transitions. Intuitively, if frictional wage dispersion is an important component of wage dispersion, then wages will not be as persistent due to the independence assumption.

We first show that the population covariance of pre-displacement and post-displacement wages depends only on the type- and economy-state specific means of these distributions and is independent of other moments of the wage offer ($F_{xs}(w)$) and pre-displacement wage distributions ($G_{xs}(w)$).

¹⁶Many models in the search literature (*e.g.* [Burdett and Mortensen 1998](#); [Postel-Vinay and Robin 2002](#)) feature relationships between firm productivity and the wage paid to workers. One way to interpret assumption A1.2 in light of these models is that the productivity of jobs within a plant may vary. The closing of a plant may reflect on the productivity of only a fraction of the jobs. This could be the case if different workers are in different labor markets with different degrees of frictions.

Let $\mu_{xs} = E_{F_{xs}}[w_{xs}^{post}]$ be the conditional mean of the job offer distribution for a worker of type x when the economy is in state s and $\Delta\mu_{xs} = E_{G_{xs}}[w_{xs}^{pre}] - \mu_{xs}$ be the difference in the conditional means of the $G_{xs}(w)$ and the $F_{xs}(w)$ distributions for a worker of type x in state s . If the post-displacement wage w_{xs}^{post} is statistically independent of the pre-displacement wage w_{xs}^{pre} , then the population covariance is

$$Cov(w^{post}, w^{pre}) = Var(\mu) + Cov(\mu, \Delta\mu). \quad (1)$$

The covariance depends only on the variation in the means of the type-specific distributions and is independent of the shape of the distributions. The derivation of Equation 1 is shown in Appendix Section A.1.

The correlation is then

$$Corr(w^{post}, w^{pre}) = \frac{Var(\mu)}{Var(w)} + \frac{Cov(\mu, \Delta\mu)}{Var(w)}$$

where $Var(w) \equiv \sqrt{Var(w^{pre})Var(w^{post})}$.¹⁷

The first term describes the variance in the means of the offer distributions across worker types (*i.e.* the between-worker variance). It thus captures differences across workers due to human capital (*e.g.* variation in ability and experience), labor market conditions (*e.g.* regional and temporal variation), and differences in the acceptance sets of jobs (*e.g.* heterogeneity in networks). Some of the persistence in wages (μ_{xs}) may be due to how worker heterogeneity interacts with search frictions through reservation wages. Whether one chooses to think of this interaction as being part of frictional wage dispersion or worker heterogeneity is one of definition. In our specification, type-specific heterogeneity in reservation wages are included in μ_{xs} . We investigate the importance of reservation wage heterogeneity in Section 3.2.¹⁸ The second term describes the covariance between the mean of the offer distribution and the difference in the means of the pre-displacement and offer distributions. This term will be non-zero if there is a correlation between μ_{xs} and average frictional wage growth. For example, the covariance could be non-zero if workers search with different intensities depending on their type as in Bagger and Lentz (2019). In Section 3.4, we find frictional wage growth to be empirically small and hence the second term will be small relative to the first term. One strength of this approach is that it does not require time-invariance of worker types and wage distributions. Hence, the wage-dispersion statistic can be calculated for different subsets of the population, at different points in the life-cycle, or different points in the business cycle to understand how the relative importance of frictional wage dispersion varies with different

¹⁷We define $Var(w)$ in this way since we find empirically that $Var(w^{CS}) \approx Var(w^F) \approx Var(w^G)$, where $Var(w^{CS})$ is the variance of wages in the cross-section and (w^F, w^G) are measured in a sample of workers who are displaced due to a plant closure. (see Section 3.1).

¹⁸If heterogeneity in reservation wages was an important factor in our setting, then we would expect to see a significant relationship between unemployment duration and pre-displacement wage, which we do not (see Table 2).

economic scenarios.

2.3 A Statistic for Frictional Wage Growth

Frictional wage growth in search models occurs through workers searching on the job. Once workers accept their first job out of unemployment, they continue to search for better jobs while they are employed. The process of finding better jobs is often called climbing the job ladder. We say that a worker exhibits frictional wage growth if their wage grows due to on-the-job search. In this section, we show how the wage-growth statistic—the fraction of displaced workers earning lower wages after unemployment—is informative of both frictional wage growth and job-to-job mobility, independent of the job offer distribution.

Measuring frictional wage growth involves estimating a measure of distance between the cross-section wage distribution ($G_{xs}(w)$) and the offer distribution ($F_{xs}(w)$). Given our main set of assumptions, the pre-displacement wage is drawn from $G_{xs}(w)$ and the post-displacement wage is drawn from $F_{xs}(w)$. The average wage loss ($E[w_{pre} - w_{post}]$) has been used in the literature to describe the distance between $G_{xs}(w)$ and $F_{xs}(w)$.¹⁹ One of our contributions to this literature is to describe a sample construction and tests where this calculation can be directly interpreted as the average frictional wage growth in the economy. We also propose a new statistic, the fraction of displaced workers earning a lower wage. The fraction earning lower wages describes more directly how far workers in the cross-section have climbed up the ladder in a way that depends less on the details of the offer distribution than the average wage loss. For example, $E[w_{pre} - w_{post}]$ can not identify dispersion in $F_{xs}(w)$ from how many offers a worker has received. In other words, the same $E[w_{pre} - w_{post}]$ can describe an economy with a disperse job offer distribution and few job offers received, or a narrow job offer distribution and many job offers received. As we will show next, the fraction of workers earning lower wages depends (given some more assumptions) only on the number of offers a worker receives and is independent of the shape of the offer distribution.

We estimate job-to-job mobility in two ways. First, making minimal assumptions about the offer arrival technology, we can use the wage-growth statistic to place bounds (*i.e.* derive an interval estimator) on the number of job offers a worker received during their last employment spell. This allows us to place a lower bound on the fraction of workers that experienced no frictional wage growth during their last employment spell. While characterizing the number of job offers a worker receives during an employment spell is not a common measure in the literature, it is more general as it makes no assumptions on the type (*e.g.* Poisson), heterogeneity (*e.g.* some firms may have higher separation rates), and time-dependence of the job offer arrival technology. Second, we also connect our wage-growth statistic to a traditional measure of job-to-job mobility commonly used in the literature. Assuming that workers receive job offers and separation shocks via Poisson arrival rates, we derive an analytical expression for the wage-growth statistic as a univariate monotone function of κ , which is the ratio of the job offer rate to the separation rate.

¹⁹See, *e.g.*, [Krolkowski \(2017\)](#).

Using the wage-growth statistic to make inference on job-to-job mobility requires three additional assumptions.

A2.1 : Workers receive job offers distributed according to $F_{xs}(w)$, independent of the worker's employment state.

There is some empirical evidence that employed workers receive better job offers compared to unemployed workers (see, *e.g.*, Faberman et al, 2017). We show later in this section that if the wage offer distribution for employed workers stochastically dominates the distribution for unemployed workers, the number of job offers and the job offer arrival rate implied by our statistic can be considered upper-bounds.

A2.2 : Wages are an order statistic of the value of the job.

In other words, workers accept any job that offers a higher wage than their current wage.²⁰ One popular type of model where this can be violated is the sequential-auction model of Postel-Vinay and Robin (2002) and Cahuc, Postel-Vinay, and Robin (2006). In Section 2.4, we show that the wage-growth statistic provides an upper bound for κ (the ratio of the job offer rate to the separation rate) in a sequential-auction model when the bargaining power of the worker is not too low.

A2.3 : Jobs are rank-preserving: If worker i at time t prefers job j to j' ($j \succ_{i,t} j'$), then $j \succ_{i,t'} j' \forall t'$.

While our analysis does not require worker types, wages, and wage distributions to be time-invariant, we assume that the *ranking* of jobs is time-invariant over an employment spell.²¹ While this might seem restrictive, this is not uncommon in the literature. Moscarini and Postel-Vinay (2013) find sufficient conditions for the existence and uniqueness of rank-preserving equilibrium in a dynamic stochastic setting. Hagedorn and Manovskii (2013) show that wages that satisfy this assumption are sufficient to explain the empirical evidence on the history-dependence of wages.²² Finally, the assumption is also consistent with how human capital accumulation (*i.e.* evolution of

²⁰Non-pecuniary benefits of a job (*i.e.* compensating differentials) also violates this assumption. In Section 4, we show how our statistic can be used to estimate a model with compensating differentials and how it is, in practice, still informative about frictional wage growth.

²¹A job may "change" within a firm-worker match if, for example, a worker receives a competing offer or the match productivity changes.

²²In particular, it is well-established that the effect of the unemployment rate at the beginning of a job spell and the minimum unemployment rate during an job spell has predictive power in a wage regression. This has been taken as evidence of history-dependence of wages. However, Hagedorn and Manovskii (2013) show that this simply masks selection of match productivities when viewed through the lens of an on-the-job search model without having to rely on sticky wages. They assume wages follow

$$\log w_t^i = \alpha \log \theta_t + \beta \log \epsilon_t^i,$$

where θ_t is a time-varying aggregate business cycle indicator and the idiosyncratic productivity, ϵ_t^i , is determined by the search process where job offers are drawn from a time-invariant distribution $\epsilon \sim F(\epsilon)$. Once frictions are controlled for by including the average labor market tightness in the employment spell and the job spell, the two unemployment rates have no predictive power.

x) is modelled in the wage growth literature.²³ At the end of this section, we extend the analysis to consider models where jobs receive match-specific productivity shocks.

Consider workers of type x who drew n independent job offers during their last employment spell and lost their job at time t . The probability $Pr_{xs}(n)$ can vary over time due to, *e.g.*, business cycle effects on the probability of receiving a job offer during the last employment spell. Since jobs are rank-preserving, the distribution of pre-displacement wages is just the maximum of n draws from the offer distribution at any time. In other words, the distribution G_{xs} is the n th order statistic of F_{xs} ,²⁴

$$G_{xs}(w|n) = F_{xs}(w)^n.$$

The fraction of workers of type x suffering a wage loss after displacement at time t is then

$$\begin{aligned} Pr_{xs}(w^{post} < w^{pre}|n) &= \int F_{xs}(w) dG_{xs}(w|n) \\ &= \int F_{xs}(w) [nF_{xs}(w)^{n-1} dF_{xs}(w)] \\ &= n \int F_{xs}(w)^n dF_{xs}(w) \\ &= n \int_0^1 z^n dz \\ &= \frac{n}{n+1}. \end{aligned} \tag{2}$$

Note that n includes the first job offer that began the previous employment spell. If a worker does not receive any additional job offers during the previous employment spell, then the probability that they will draw a lower wage after displacement is $1/2$. Importantly, notice that the fraction of workers earning lower wages after displacement depends only on the number of job offers they receive (n) and is independent of the wage offer distribution, $F_{xs}(w)$. This is because the probability $Pr_{xs}(w^{post} < w^{pre})$ depends only on the order statistic of w^{pre} and not the actual value of the wage. In other words, conditional on n , the probability of wage loss is independent of worker type and how wage distributions evolve over time: $Pr_{xs}(w^{post} < w^{pre}|n) = Pr(w^{post} < w^{pre}|n)$.

So far the only assumption made regarding the job arrival technology is that job offers are drawn from $F_{xs}(w)$. Without making further assumptions on the offer arrival technology, we can use the observed fraction of displaced workers earning lower wages to place bounds on the fraction of workers who received n offers during their last employment spell. The fraction of displaced

²³See, *e.g.*, Bagger, Fontaine, Postel-Vinay, and Robin (2014), Yamaguchi (2010), Burdett, Carrillo-Tudela, and Coles (2011), and Bowlus and Liu (2013).

²⁴This is where the rank-preserving jobs assumption is needed.

workers earning lower wages can be written as a weighted sum

$$\begin{aligned} Pr_{xs}(w^{post} < w^{pre}) &= \sum_{n=1}^{\infty} Pr(w^{post} < w^{pre}|n) Pr_{xs}(n), \\ &= \sum_{n=1}^{\infty} \frac{n}{n+1} Pr_{xs}(n), \end{aligned} \quad (3)$$

where $Pr_{xs}(n)$ represents the probability that a worker of type x received n offers during an employment spell and must satisfy both $Pr_{xs}(n) \geq 0$ and $\sum_{n=1}^{\infty} Pr_{xs}(n) = 1$. $Pr_{xs}(n)$ will depend on the details of how workers receive job offers and is determined by the offer arrival technology, which we have made minimal restrictions on so far. While we cannot derive a point estimator of $Pr_{xs}(n)$ without further assumptions, we can use Equation 3 to place bounds on $Pr_{xs}(n)$. For example, if the estimated wage-growth statistic is less than $2/3$,²⁵ the proportion of workers of type x who received n offers during their last employment spell is

$$Pr_{xs}(n) \in \begin{cases} \left[4 - 6\hat{Pr}_{xs}, 2 - 2\hat{Pr}_{xs} \right] & \text{for } n = 1, \hat{Pr}_{xs} \leq 2/3 \\ \left[0, \frac{n+1}{n-1}(2\hat{Pr}_{xs} - 1) \right] & \text{for } n > 1, \hat{Pr}_{xs} \leq 2/3, \end{cases} \quad (4)$$

where $\hat{Pr}_{xs} \equiv \hat{Pr}_{xs}(w^{post} < w^{pre})$ is the estimated wage-growth statistic when the economy is at state s_t for worker type x .²⁶ These bounds can be used to place empirical limits for different types of models. For example, $4 - 6\hat{Pr}_{xs}$ describes a lower bound for the fraction of workers who experience no frictional wage growth in the previous employment spell. These bounds can easily be extended to a labor market with involuntary job offers or match-specific productivity shocks, where $Pr_{xs}(n)$ is then the number of job offers since the most recent unemployment spell *or* involuntary job offer *or* match-specific productivity shock.²⁷ One of the benefits of focusing on the number of job offers ($Pr_{xs}(n)$) is that we do not need to take a stand about the offer arrival technology and how the arrival technology varies over time.

While inference on the number of job offers received ($Pr_{xs}(n)$) is relatively general, it is the first time, to our knowledge, that it has been used to describe job-to-job mobility. In order to be able to compare our results to measurements in the literature, we relate the frictional wage-growth statistic to $\kappa \equiv \lambda^e/\delta$, which is the ratio between the job offer arrival rate and the job destruction rate. To do this, we need to make an assumption on the job offer arrival technology. Following the literature, we assume that workers receive job offers while employed at a constant Poisson

²⁵Different bounds can be calculated if $\hat{Pr}_{xs}(w^{post} < w^{pre}) > 2/3$, but we do not find evidence for that empirically.

²⁶The bounds are calculated by setting the remaining probability to load entirely on extreme values of n . For example, the lower bound for $Pr(1)$ occurs when $Pr(2) = 1 - Pr(1)$ and $Pr(m) = 0 \quad \forall m > 2$. The lower bound can then be found by solving for $Pr(1)$ using Equation 3: $\hat{Pr} = \frac{1}{2}Pr(1) + \frac{2}{3}[1 - Pr(1)]$. Likewise, the upper bound can be found by solving $\hat{Pr} = \frac{1}{2}Pr(1) + 1[1 - Pr(1)]$.

²⁷This assumes that match-specific productivity shocks are drawn from $F_{xs}(w)$.

rate of λ^e and lose their jobs at a constant Poisson rate δ .²⁸ These two parameters determine the probability distribution of the number of job offers a worker receives during an employment spell. Specifically, the probability of receiving $n - 1$ additional job offers before a separation shock is²⁹

$$\begin{aligned} Pr(n) &= \left(\frac{\lambda^e}{\lambda^e + \delta} \right)^{n-1} \frac{\delta}{\lambda^e + \delta} \\ &= \left(\frac{\kappa}{\kappa + 1} \right)^{n-1} \frac{1}{\kappa + 1}. \end{aligned} \quad (5)$$

Differences in the number of job offers a worker receives will depend on their type x . It is important to note here, that the goal of this analysis is not to estimate κ_x *per se*, but to relate our statistic to a measure of frictional wage growth commonly used in the literature. In other words, viewed through the lens of a search model with constant Poisson arrival rates, what is the inferred κ_x from the wage-growth statistic? The relationship between the wage-growth statistic and κ_x is

$$\begin{aligned} Pr_x(w^{post} < w^{pre}) &= \frac{1}{\kappa_x + 1} \sum_{n=1}^{\infty} \frac{n}{n+1} \left(\frac{\kappa_x}{\kappa_x + 1} \right)^{n-1} \\ &= 1 - \frac{(\kappa_x + 1) \ln(\kappa_x + 1) - \kappa_x}{\kappa_x^2}. \end{aligned} \quad (6)$$

The derivation of Equation 6 is found in Appendix Section A.2.³⁰

The one-to-one relationship between the wage-growth statistic and κ_x is depicted in Figure 1. As $\kappa_x \rightarrow \infty$ (*e.g.* $\lambda_x^e \rightarrow \infty$), $Pr_x(w^{post} < w^{pre}) \rightarrow 1$. As the rate of on-the-job offers increases relative to the job destruction rate, workers, on average, climb further up the job ladder before suffering displacement. The probability that they then earn a lower wage in their first draw from $F_{xs}(w)$ becomes very high. Likewise as $\kappa_x \rightarrow 0$ (*e.g.* $\lambda_x^e \rightarrow 0$), $Pr_x(w^{post} < w^{pre}) \rightarrow 1/2$. In other words, as workers become more likely to get a job destruction shock relative to a job offer in their first job, the probability goes to 1/2.

Notice, that any population measure of the probability, $Pr(w^{post} < w^{pre})$, is going to be a weighted sum over the type probabilities, $\sum_x \pi_x Pr_x(w^{post} < w^{pre})$, where π_x is the fraction of workers of type x .

²⁸The search literature on frictional wage growth typically assumes time-invariant Poisson arrival rates.

²⁹From the mathematics literature on Poisson processes (*e.g.* Gallager 2013), the probability that the k^{th} arrival of process 1 occurs before the j^{th} arrival of process 2 is

$$Pr(S_k^1 < S_j^2) = \sum_{i=k}^{k+j-1} \binom{k+j-1}{i} \left(\frac{\lambda_1}{\lambda_1 + \lambda_2} \right)^i \left(\frac{\lambda_2}{\lambda_1 + \lambda_2} \right)^{k+j-1-i}.$$

Note that this assumes that (employment) spells are not censored. For example, if κ is estimated using the wage-growth statistic for young workers with less than five years of labor market experience, employment spells longer than five years would be censored and estimates of κ using Equation 6 could be biased downwards.

³⁰We show that the result can also be derived using Burdett and Mortensen (1998) steady-state accounting arguments in online Appendix Section C.

Different Offer Distribution If the wage offer distribution of employed workers stochastically dominates the distribution of unemployed workers ($F_{xs}^e(w) \leq F_{xs}^u(w) \forall w$), then the mobility parameters implied by the wage-growth statistic derivations can be considered upper bounds for the true mobility parameters. In this case, the distribution $G_{xs}(w|n) = F_{xs}^u(w)F_{xs}^e(w)^{n-1}$. The derivation of Equation 2 is similar: $Pr_{xs}(w^{post} < w^{pre}|n) = \int F_{xs}^u(w)dG_{xs}(w|n) = 1 - \int G_{xs}(w|n)dF_{xs}^u(w) = 1 - \int F_{xs}^u(w)F_{xs}^e(w)^{n-1}dF_{xs}^u(w) \geq 1 - \int F_{xs}^u(w)^ndF_{xs}^u(w) = \frac{n}{n+1}$. Giving us $\frac{n}{n+1} \leq Pr_{xs}(w^{post} < w^{pre}|n)$.

Involuntary Job Offers and Match-Specific Productivity Shocks Our result can be extended to cases where workers receive involuntary job offers or where jobs receive match-specific productivity shocks. An involuntary job offer is a job offer that employed workers must accept or go into unemployment. These have in recent years become common in empirical search models, see e.g. [Bagger and Lentz \(2019\)](#) or [Taber and Vejin \(2020\)](#). "Involuntary job offers" can represent many different situations, but one common interpretation is that it is a situation where a worker receives an advanced layoff notice and found a job prior to getting fired. Analogously, consider a model with match-specific productivity inspired by [Mortensen and Pissarides \(1994\)](#), but where we modify the model with two realistic features. First, the initial match-productivity is drawn from the offer distribution and there is on-the-job search. Let λ_x^d represent the total Poisson rate of involuntary job offers and match-specific productivity shocks. The derivation is the same as before, except that we need to calculate the probability of receiving $n - 1$ additional job offers before receiving a job separation, an involuntary job offer, or match-specific productivity shock,

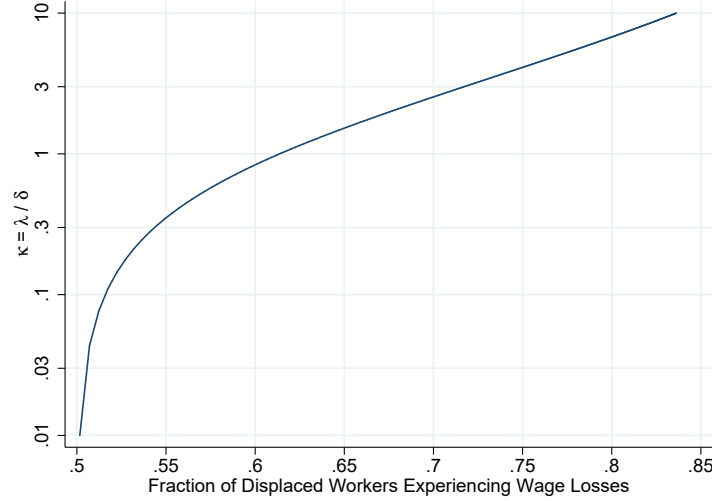
$$Pr_x(n) = \left(\frac{\lambda_x^e}{\lambda_x^e + \delta_x + \lambda_x^d} \right)^{n-1} \frac{\delta_x + \lambda_x^d}{\lambda_x^e + \delta_x + \lambda_x^d}.$$

The relationship between the wage-growth statistic and the Poisson parameters in a model with involuntary job offers is

$$Pr_x(w^{post} < w^{pre}) = 1 - \frac{(1 + \tilde{\kappa}_x) \ln(1 + \tilde{\kappa}_x) - \tilde{\kappa}_x}{\tilde{\kappa}_x^2},$$

where $\tilde{\kappa}_x = \frac{\lambda_x^e}{\delta_x + \lambda_x^d}$. In terms of the job ladder, an involuntary job offer or match-specific productivity shock functions in a similar way to a job destruction shock, in that workers lose their search capital.

Figure 1: Relationship between fraction of workers earning lower wages after displacement and κ



2.4 Extension to Sequential-Auction Models

Sequential-auction models are a class of search models that add between-employer competition to the canonical wage-ladder model.³¹ Unemployed workers meet with an employer and bargain over the wage, where unemployment is the worker's outside option. When employed workers consider a new job offer, their outside option is their current employer. The wage then depends not only on the highest offer received, but also on the second highest offer. Workers draw job offers from an offer distribution $F_x(p)$, where p is the flow productivity. Thus, the highest wage an employer can pay to a worker of type x and still earn non-negative profits is p_x . Note that p_x is capturing the employer heterogeneity for workers of type x . The wage in these models is described by

$$w_x(p_1, p_2) = p_1 - (1 - \beta_x) \int_{p_2}^{p_1} \frac{\rho_x + \lambda_x \bar{F}_x(z)}{\rho_x + \lambda_x \beta_x \bar{F}_x(z)} dz, \quad (7)$$

where β_x is the bargaining power of the worker, $\bar{F}_x(z) = (1 - F_x(z))$ where $F_x(z)$ is the job offer distribution. The discount factor ρ_x includes the worker's time discounting and Poisson rates for mechanisms that result in a worker leaving the job involuntarily (separation rates, involuntary job offers, etc). The wage depends on both the highest offer (p_1) and the second highest offer (p_2) received during the last employment spell. The sequential-auction model nests the wage-ladder model from the previous section when $\beta_x = 1$, where wages are simply equal to p_1 .

There are two elements of the sequential-auction model that complicates the calculation of the wage-growth statistic and κ_x : rent-sharing and expectations of future wage growth. To see this,

³¹See [Postel-Vinay and Robin \(2002\)](#) and [Cahuc, Postel-Vinay, and Robin \(2006\)](#) for seminal contributions.

we re-write Equation 7 in terms of a static component and an option value component

$$w_x(p_1, p_2) = \underbrace{\beta_x p_1 + (1 - \beta_x) p_2}_{\text{Static Component}} - \underbrace{(1 - \beta_x)^2 \int_{p_2}^{p_1} \frac{\lambda_x \bar{F}_x(z)}{\rho_x + \lambda_x \beta_x \bar{F}_x(z)} dz}_{\text{Option Value Component}}. \quad (8)$$

The static component reflects the bargaining between the worker and the firm over the surplus of the match (rent-sharing) ignoring the value of future wage growth (*i.e.* option value). The option value component reflects the wage the worker is willing to give up because of the expected future wage growth. Unfortunately, it is not possible in this case to express $Pr_x(w^{post} < w^{pre})$ independently of the offer distribution, as we do in Equation 2. This is both due to the weighted average between the highest and second highest offer in the static component and the additional integral in the option value component.

The goal of this sub-section is to find sufficient conditions such that, given the same environment, the sequential-auction model will generate a weakly higher probability of wage loss than the wage-ladder model. We can then interpret our inferred number of job offers or κ_x from the wage-ladder model as upper bounds for the sequential-auction model when $\beta_x < 1$. Since our estimates of the implied κ are low, an upper bound does not affect the interpretation of our estimates.

Let p_1^{pre} and p_2^{pre} represent the highest and second highest offers (in terms of p_x) from the pre-displacement employment spell. Let p_1^{post} represent the first offer out of unemployment. When $\beta_x = 1$, $p_1^{pre} > p_1^{post}$ will result in a wage loss and $p_1^{pre} < p_1^{post}$ will result in a wage gain. Our approach is to derive conditions on β_x such that $p_1^{pre} > p_1^{post}$ will always result in $w^{pre} > w^{post}$ in the sequential-auction model. These are sufficient conditions as we will be comparing workers with the same history of job offers and asking if a wage loss in the wage-ladder model ($\beta_x = 1$) will result in a wage loss in the sequential-auction model ($\beta_x < 1$). Necessary conditions would only require more wage losses on average, rather than point-wise for each possible offer history.

It is easy to see from Equation 8, that if the option value component of wages is small (*e.g.* $\lambda_x \ll \rho_x$), then $w(p_1^{pre}, p_2^{pre}) > w(p_1^{post}, b_x)$ if $p_1^{pre} > p_1^{post}$ and $p_2^{pre} \geq b_x$ with b_x being the flow value of unemployment. More generally, we can derive a sufficient condition on the bargaining power (β_x), such that the sequential-auction model will result in weakly more wage losses compared to the wage-ladder model ($\beta_x = 1$) even when the option value is important.

Proposition 2.1. *Consider a job offer history for a worker of type x , where the highest pre-displacement offer is p_1^{pre} , the second-highest pre-displacement offer is at least as large as the flow value of unemployment $p_1^{pre} \geq p_2^{pre} \geq b_x$, and the post-displacement offer is p_1^{post} . If $p_1^{post} < p_1^{pre}$ and $\beta_x > \frac{\lambda_x}{2\lambda_x + \rho_x}$, then $w(p_1^{post}, b_x) < w(p_1^{pre}, p_2^{pre})$.³²*

We have shown that when $\beta_x > \frac{\lambda_x}{2\lambda_x + \rho_x}$, the sequential-auction model predicts wages losses whenever there are wage losses for $\beta_x = 1$, independent of the offer distribution. In this case, we can consider the implied κ_x from the $\beta_x = 1$ model as an upper bound for the implied κ_x in

³²The proof is in Appendix Section A.3.

sequential-auction models with $\frac{\lambda_x}{2\lambda_x + \rho_x} < \beta_x < 1$. In Section 4, we estimate a basic wage-ladder model. If we calculate the sufficient condition on β_x for the estimated model we get 0.307.³³ In the literature β_x is often found to be in the range of 0.2-0.4.³⁴ Note that the lower bound on β_x is a loose lower bound and we conjecture that the implied κ_x from the wage-ladder model is an upper bound for the sequential-auction model in most cases. This is supported by a numerical example in Appendix Section F.4, where we show that the implied κ from the wage-ladder model is an upper bound for any value of β in our estimated model and not just values that satisfy Proposition 2.1.³⁵

3 Quantitative Implications for Frictional Wage Growth and Wage Dispersion

In this section, we present estimates of the two statistics discussed. We start by describing the two datasets in Section 3.1. In Section 3.2, we check the robustness of the identifying assumptions empirically. Section 3.3 discusses the effect of measurement error and derives simple corrections for both statistics. Section 3.4 presents and discusses the estimated statistics and their implications.

3.1 Data

We estimate the frictional wage dispersion and frictional wage-growth statistics using two US surveys: the Displaced Worker Supplement to the Current Population Survey (CPS-DWS) and the Survey of Income and Program Participation (SIPP). We repeat the analysis in two different datasets for a number of reasons. First, each dataset has its strengths and weaknesses. The SIPP records wages each month, but does not ask about plant closures. The CPS-DWS identifies plant closures (our preferred characterization of exogenously displaced workers), but measures post-displacement wages up to three years after starting the post-displacement job. A large delay in the measurement of wages is clearly not ideal, since we want to measure the wage immediately after finding the first job. Second, using both surveys we are able to show that our estimates are robust to survey design, time period covered, definition of displaced workers, and when and how wages are measured. The goal is to define displacements as involuntary exogenous separations based on the operating decisions of the employer, such as firm/plant closings and permanent layoffs. Other

³³ $\beta_x > \frac{0.173}{2 \times 0.173 + 0.05 + 0.116 + 0.052}$

³⁴ Bagger, Fontaine, Postel-Vinay, and Robin (2014) find β_x to be around 0.3 across all educational groups, while Cahuc, Postel-Vinay, and Robin (2006) find that bargaining power is increasing in the observable ability of the worker with some differences across sectors. The bargaining power of managers is on average around 0.45, while it is on average 0.05 for low skilled workers. Bagger and Lentz (2019) finds β to be 0.231.

³⁵ The necessary conditions likely require some restrictions on the offer distribution as we have found that it is possible to construct an offer distribution where the wage-ladder model has more wage losses than a model with $\beta = 0$. One such distribution has two mass points, one at $p = b$ and another at $p > b$. This counter-example is an economy where a significant fraction of workers are paid wages at or below the flow value of unemployment, which seems to be at odds with empirical observations.

types of separations—*e.g.* due to quits or being fired with cause—are not included, since these may be endogenous, *e.g.* to individual wages. The empirical analysis studies prime-aged (25-54 years old), full-time (at least 35 hours/week), private sector workers who are not working in agriculture or construction. All earnings are in 2010 US dollars (deflated by the CPI).

Description of SIPP The Survey of Income and Program Participation (SIPP) is a continuous series of short panels.³⁶ Our analysis includes the 1996, 2001, 2004, and 2008 panels.³⁷ The duration of each panel ranges from three to four years. Each individual is surveyed once every four months and is asked about their employment in each month during the previous four months. In particular, if they leave a job they are asked for the reason. In the SIPP, we characterize a displaced worker as a worker who left an employer (and did not return) for one of three reasons: "Layoff," "Employer bankrupt," and "Slack work or business conditions." Unfortunately, workers are not asked about plant closings and separations due to a bankrupt employer are a rare occurrence.

We construct the sample of displaced workers in SIPP via the following steps. The main displaced worker sample consists of prime-aged, full-time, private-sector workers who were displaced at least one year before the last wave of the panel. This is the job-unemployment or "JU" sample. In order to avoid using the earnings for a month in which the worker was not fully employed, we use the last reported monthly wage earnings in the wave previous to displacement as the pre-displacement wage.³⁸ The job-unemployment-job or "JUI" sample additionally selects displaced workers who find a full-time, private-sector job within a year of displacement. We choose one year because we want avoid considerations about human capital changing during the unemployment spell. The post-displacement wage is the first reported monthly wage earnings of the post-displacement job in the wave after displacement. In addition, we construct a cross-section of prime-aged, full-time, private-sector workers that can be compared to the pre-displacement workers in order to investigate the representativeness of displaced workers. This is the cross-section or "CS" sample. For each individual in the "CS" sample, we flag months when they were prime-aged and employed in a full-time private-sector job. Then for each individual, we randomly select a flagged month and record the monthly earnings. We only include observations at least one year before the last wave of the panel to match the selection of the displacement sample. Descriptive statistics of the SIPP sample are reported in Appendix Table 1.

³⁶We use the Center for Economic and Policy Research SIPP Uniform Extracts: *Center for Economic and Policy Research. 2014. SIPP Uniform Extracts, Version 2.1.7. Washington, DC.*

³⁷Earlier panels did not ask detailed questions about separations, so we can not identify displaced workers in those datasets.

³⁸For workers who are paid an hourly wage, the monthly wage earnings are calculated by multiplying the hourly wage times the hours the worker reported. We do not use income that has been imputed. We winsorize monthly income between \$640 and \$14,500 in 2010 US dollars (deflated by the CPI). The upper limit drops top-coded earnings as recommended by the Center for Economic and Policy Research. Our results are robust to using the panel-specific top-codes (\$12,500 in nominal dollars for 1996 and 2001; and \$16,666 in nominal dollars for 2004 and 2008). The wage-growth statistic estimates are also robust to including individuals who are top-coded in one, but not both wage measurements (the case when wage comparisons can still be made).

Description of CPS-DWS The Displaced Workers Survey (CPS-DWS) is a biennial supplement to the CPS taken during the January or February data collection.³⁹ Our CPS-DWS sample includes all surveys taken between 1984 and 2016. The CPS-DWS asks respondents if they had experienced a displacement in the last 3 years and what year they lost their job.⁴⁰ The data have information on worker demographics; occupation, industry, and weekly earnings at the pre-displacement job; weeks without work after displacement; and occupation, industry, and weekly earnings at the current job.⁴¹ We also use data from the outgoing rotation group (CPS-ORG) supplement from January two years before each CPS-DWS survey in order to make comparisons between the displaced worker’s pre-displacement job and the jobs of workers from the cross-section. The Outgoing Rotation Group (CPS-ORG)—also called the earner study—asks about usual weekly hours and earnings in one fourth of the households surveyed each month.⁴²

We restrict the CPS-DWS sample to workers who were displaced from a full-time private-sector job by a plant closing. We keep the workers who report being displaced “two years” before the survey to minimize both selection bias and post-displacement wage growth bias.⁴³ This is the “JU” sample for the CPS-DWS. We then require that they are reemployed at the survey date at a different full-time private-sector job (*i.e.* they were not recalled). This is the “JUI” sample for the CPS-DWS. We restrict our analysis to full-time jobs because the CPS-DWS only provides “usual” weekly earnings and the full/part time status of the worker’s old job before 1994, hence it is difficult to control for hours of work beyond requiring full-time status in a consistent way across the years. Finally, we use the CPS-ORG sample to construct a “CS” sample that can be compared to the CPS-DWS’s “JU” sample. To each CPS-DWS survey, we append the CPS-ORG sample

³⁹The CPS-DWS was given in February between 1994 and 2000 and was given in January all other years.

⁴⁰For example, the 2016 supplement asked, “During the last 3 calendar years, that is, January 2013 through December 2015, did (name/you) lose a job, or leave one because: (your/his/her) plant or company closed or moved, (your/his/her) position or shift was abolished, insufficient work or another similar reason?” Before 1994, the CPS-DWS asked about layoffs in the last 5 years. To keep the sample consistent, we drop observations reporting layoffs more than 3 years in the past. If the worker experienced more than one layoff in the past three years, they ask about the job they held the longest.

⁴¹For workers who are paid an hourly wage, the weekly wage earnings are calculated by multiplying the hourly wage times the hours the worker reported. We winsorize weekly income between \$160 and \$2,600 in 2010 US dollars (deflated by the CPI). The upper limit drops top-coded earnings in a similar way to the SIPP sample selection. Our results are robust to using the wave-specific top-codes (\$1,923 in nominal dollars before 1997; and \$2884.61 in nominal dollars afterwards). The wage-growth statistic estimates are also robust to including individuals who are top-coded in one, but not both wage measurements (the case when wage comparisons can be made). We also note that about 9 percent of displaced workers report the same pre- and post-displacement wages. Dropping these workers would lower both the wage-dispersion statistic (they increase the correlation) and the wage-growth statistic (they are classified as wage losses in real terms).

⁴²Households in the CPS are surveyed for four successive months, not surveyed for eight months, and then surveyed for four successive months. The CPS-ORG survey occurs in the fourth and eighth interview (fourth and sixteenth month).

⁴³Workers are asked if they last worked at the lost job either “last year,” “two years ago,” or “three years ago.” We found that workers who reported “last year” and had a job at the time of the interview are strongly selected compared to workers who report losing their job “two” or “three” years ago. The “last year” workers had shorter nonemployment durations and higher wages in the post-displacement job. We do not include the workers who report being displaced three years ago as they have been working at least a year longer and have higher post-displacement wages due to post-displacement wage growth.

from exactly two years earlier. The CPS CS sample includes data from 1996-2014.⁴⁴ Descriptive statistics of the sample are reported in Appendix Table 2.

3.2 Robustness of Identifying Assumptions

Representativeness of Displaced Workers One important threat to our identification strategy is that workers with certain wages may be more likely to be displaced. For example, workers at a low-paying plant may have a higher risk of displacement. Another example is that low quality worker-firm matches may be more sensitive to productivity shocks that lead to separations. These mechanisms will bias the wage-growth statistic downwards and also make the wage-dispersion statistic difficult to interpret. It is well-known in the displacement literature that wages at closing plants are low.

Table 1 shows a series of regressions comparing the pre-displacement wages of displaced workers to the wages of CS workers in both the CPS and the SIPP. In both the SIPP and CPS, the pre-displacement wages of displaced workers are significantly below the wages of the cross section of workers (column 1). Specifically, we find that pre-displacement wages are seven to eight log points less than the average worker in the US economy.

While our results on pre-displacement wages are consistent with the displacement literature, we find that the differences in wages vary quite a bit if we separate the workers who find a job within a year and those that do not.⁴⁵ In columns 2-4 in Table 1, we separate displaced workers who do not find a full-time, private-sector job within a year (JU) and those that did (JUU). In the SIPP in panel A, the JUU workers are much less selected compared to the CS workers (column 2) and the difference is not statistically significant. Once we control for four education indicators (column 3), the JUU-CS difference becomes even smaller. The similarity between JUU and the CS samples is robust to adding a rich set of controls (column 4), (*e.g.* demographics, occupation, tenure, experience, year hired, etc). While it is possible that these workers are positively selected in some unobserved characteristic, this characteristic would have to be orthogonal to the set of rich observables that we consider. We find similar results in the CPS. Thus, the wage differences of displaced workers are being driven by workers who do not find full-time private-sector jobs within a year after displacement and are not in the sample we use to calculate the statistics. Table 3 in the Appendix repeats specification (4) for sex, education, experience, tenure, and occupation

⁴⁴The goal of constructing the CS sample is to be able to compare, along multiple dimensions, the displaced workers to workers in the cross section. One important dimension is job tenure. We do not include earlier years in the CS Sample as the Job Tenure Supplement was not given before 1996.

⁴⁵The requirement that workers find a job *within one year* does not appear to be important. The SIPP panels are too short to follow workers for much more than a year after displacement. We can compare workers in the CPS-DWS who were displaced "two years ago" vs. "three years ago." Surprisingly we find that both the fraction of workers who find a job and the pre-displacement wage is almost identical in the two samples. For example, 52 percent of the "two years ago" sample and 54 percent of the "three years ago" had a full-time private-sector job at the time of the interview, even though the "three years ago" workers had an additional year to find a job. Only 42 percent of "last year" workers had a job at the time of the interview.

Table 1: Comparing Pre-displacement Wages of Displaced Workers to the Cross Section

Panel A: Log Wage Regressions								
	SIPP				CPS			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
Displaced	-0.0832*** (0.0096)				-0.0788*** (0.0125)			
JUJ		-0.0224 (0.0140)	0.0007 (0.0124)	0.0083 (0.0105)		-0.0167 (0.0171)	-0.00994 (0.0152)	0.00630 (0.0134)
JU		-0.133*** (0.0128)	-0.0832*** (0.0113)	-0.0626*** (0.0096)		-0.147*** (0.0179)	-0.0935*** (0.0159)	-0.0605*** (0.0141)
Year FE	X	X	X	X	X	X	X	X
Education			X	X			X	X
Demographics				X				X
Labor History				X				X
R^2	0.005	0.006	0.221	0.446	0.002	0.003	0.212	0.393
N	95236	95236	95236	95236	59986	59986	59986	59986
Panel B: Kolmogorov-Smirnov Tests of Equality of JUJ and CS Wage Distributions								
	All	HS Dropout	HS Graduate	Some College	College Graduate			
SIPP p -val	0.021	0.086	0.030	0.459	0.076			
SIPP N	92909	8990	28275	32455	23189			
CPS p -val	0.145	0.383	0.151	0.294	0.634			
CPS N	59087	4638	18788	17448	18213			

Note: Standard errors are in parenthesis. Earnings of displaced workers are compared to a cross-section (CS) of workers. Earnings are measured as log of weekly (CPS-DWS) or monthly (SIPP) wage earnings. Samples consist of prime aged (25-54 years old), full-time, private-sector workers who are not working in agriculture or construction. Displaced is an indicator for workers who were displaced from their job (SIPP: layoff, employer bankruptcy, or slack work conditions; CPS: plant closing). JUJ is an indicator for displaced workers who found a full-time private-sector job within a year and JU is an indicator for displaced workers who did not find a full-time private-sector job within a year. Education includes four education indicators. Demographics includes race and sex indicators. Labor History includes indicators for experience, tenure, occupation, and year the worker started the job. CPS Data combines pre-displacement weekly earnings of workers from the 1998-2016 Displaced Worker Surveys (CPS-DWS) with weekly earnings of a cross-section of workers who were in the Outgoing Rotation Groups (CPS-ORG) two years before each Displaced Worker Survey. SIPP combines pre-displacement earnings of displaced workers with one randomly chosen wage observation for each cross-section worker from the 1996, 2001, 2004, and 2008 panels. See Section 3.1 and Appendix Section E for more details. *** $p < 0.001$

sub-samples and confirms the findings in Table 1.

We interpret these findings on pre-displacement wages as indicating that there are two broad types of displaced workers. The first type have marketable skills and were paid similar wages to other workers in the economy. The first type finds a new job quickly (*i.e.* less than a year). The second type of displaced worker is employed using obsolete/unmarketable skills. The pre-displacement firm can pay low wages to these workers and not worry about losing them to other firms. Once they are displaced they have a difficult time finding a new full-time private-sector job. Our analysis compares pre-displacement wages with post-displacement wages, and hence focuses on the first type of worker. Finally, since our statistics are not linear in the pre-displacement wages, we care about the full distribution and not just the mean. In panel B of Table 1, we present results from a series of Kolmogorov-Smirnov tests of the equality of the JUJ and CS wage distributions. We find that in the CPS data we cannot reject unconditional equality, while in the SIPP we can at the 5 percent significance level. Once conditioning on education by testing the within education distributions, we cannot reject equality in any of the educational samples using SIPP or CPS.⁴⁶ Not only are the means the same for the JUJ and CS samples, but, conditional on education, the distributions are also similar.

Independence of Pre- and Post-displacement Wages There are a number of economic mechanisms that could generate a correlation between the pre-displacement wage and the worker’s reservation wage, violating the independence assumption. A few examples are savings, loss aversion, option to be recalled to previous job, and unemployment benefits. First, higher wages may lead workers to have higher savings. Higher savings may lead to a higher reservation wage when unemployed due to a better ability to smooth consumption. Second, workers may have loss aversion, where there is a direct utility cost of accepting a lower wage relative to the wage prior to unemployment. Third, workers may have the option to be recalled or expect the option to be recalled to their pre-displacement job, and, hence, will have a higher reservation wage. Finally, unemployment benefits depend on a worker’s pre-displacement wage up to a maximum benefit that varies by state. All four of these mechanisms lead high-wage workers to have higher reservation wages and, conditional on worker type, longer unemployment durations.

We test the prediction that higher pre-displacement wages lead to longer unemployment durations in the SIPP and the CPS-DWS. In the CPS-DWS, displaced workers are asked how many weeks they were without work. In the SIPP, workers report the calendar month and year they start and finish working at a job. We use the start and end dates to calculate the number of months between the end of the pre-displacement job and the start of the post-displacement job. Table 2 shows a series of regressions of the unemployment duration measured in days on the pre-displacement log wage including different sets of controls.⁴⁷ The top panel shows the results for the SIPP JUJ sample and the bottom panel shows the results for the CPS-DWS JUJ sample. In all cases, we do

⁴⁶The HS Graduate comparison in the SIPP sample is weakly statistically significant with a p -value of 0.03.

⁴⁷We have also done the regressions using the log to unemployment duration. The qualitative results are very similar.

not find evidence of a relationship between unemployment duration and the pre-displacement log wage. Table 4 in the Appendix repeats specification (4) for sex, education, experience, tenure, and occupation sub-samples and confirms the findings in Table 2. We thus conclude that mechanisms that violate the independence assumption by generating a positive correlation between pre- and post-displacement wages through higher reservation wages are not a major concern.

Table 2: Relationship between Unemployment Duration and Pre-Displacement Wage

	Unemployment Duration (days)			
	(1)	(2)	(3)	(4)
SIPP Log Wage	-5.040 (3.843)	-1.448 (4.533)	2.255 (4.727)	2.156 (5.320)
R^2	0.023	0.029	0.034	0.089
N	1914	1914	1914	1914
CPS-DWS Log Wage	-4.566 (5.409)	2.068 (5.860)	4.074 (6.124)	-8.523 (6.717)
R^2	0.042	0.047	0.051	0.117
N	2062	2062	2062	2062
Year FE	X	X	X	X
Education		X	X	X
Demographics			X	X
Labor History				X

Note: Standard errors are in parenthesis. Each element in the table is a regression of the unemployment duration (measured in days unemployed) on the pre-displacement log wage. Log wages are measured as log of weekly (CPS-DWS) or monthly (SIPP) wage earnings. The unemployment duration in the SIPP is measured as the number of months between the pre-displacement job and the post-displacement job. The unemployment duration in the CPS is the number of weeks without work reported by the worker. The number of observations for the CPS-DWS sample do not match the analysis sample as not everyone answers the unemployment duration question. Demographics controls include race and sex indicators. Education controls include high school graduate, some college, and college graduate indicators. Labor History includes indicators for potential experience, tenure at lost job, occupation at lost job, and the year the worker started the lost job.

3.3 Measurement Error in Wages

If wages are measured with classical measurement error, the wage-dispersion statistic will be biased towards zero and the wage-growth statistic will be biased towards 1/2. In this section, we describe

simple measurement-error corrections for each statistic.⁴⁸

A correction for the wage-dispersion statistic can be done by dividing the correlation between pre-displacement and post-displacement wages by the reliability (or signal-to-total-variance) ratio of wages (λ_w^{rel}). Assume that wages have classical measurement error

$$w = \eta_w + \nu_w,$$

where η_w is the true wage and ν_w is the measurement error in wages. Let measurement error be independent between pre-displacement and post-displacement measurements and also independent of true wages. The correlation between pre-displacement and post-displacement wages is then

$$\begin{aligned} corr(w^{pre}, w^{post}) &= \frac{cov(\eta_w^{pre}, \eta_w^{post})}{\sigma_{w^{pre}} \sigma_{w^{post}}} \\ &= \lambda_w^{rel} corr(\eta_w^{pre}, \eta_w^{post}), \end{aligned}$$

where we assume that the reliability ratio ($\lambda_w^{rel} = \frac{\sigma_{\eta_w}^2}{\sigma_{\eta_w}^2 + \sigma_{\nu_w}^2}$) is the same for pre- and post-displacement wages ($\lambda_{w^{pre}}^{rel} = \lambda_{w^{post}}^{rel}$). Hence, a simple measurement-error correction is to divide the measured wage-dispersion statistic by the reliability ratio.

The wage-growth statistic can be corrected for measurement error by doing a simple deconvolution. Let the difference in pre-displacement and post-displacement wages be

$$\Delta w = \eta_{\Delta w} + \nu_{\Delta w},$$

where $\eta_{\Delta w}$ is the true difference in log-wages and $\nu_{\Delta w}$ is classical measurement error. Assume that $\eta_{\Delta w}$ and $\nu_{\Delta w}$ are independent and normally distributed, where the measurement error has mean zero ($\mu_{\nu_{\Delta w}} = 0$). Hence, Δw is also normally distributed with mean $\mu_{\Delta w} = \mu_{\eta_{\Delta w}}$ and variance $\sigma_{\Delta w}^2 = \sigma_{\eta_{\Delta w}}^2 + \sigma_{\nu_{\Delta w}}^2$. We can express the true fraction earning lower wages ($\Pr(\eta_{\Delta w} < 0)$) in terms of the fraction measured (with classical measurement error) in the data ($\Pr(\Delta w < 0)$):

$$\begin{aligned} \Pr(\eta_{\Delta w} < 0) &= \Phi \left[\frac{-\mu_{\eta_{\Delta w}}}{\sigma_{\eta_{\Delta w}}} \right] = \Phi \left[\frac{-\mu_{\Delta w}}{\sigma_{\Delta w} \sqrt{\lambda_{\Delta w}^{rel}}} \right] \\ &= \Phi \left[\frac{\Phi^{-1} [\Pr(\Delta w < 0)]}{\sqrt{\lambda_{\Delta w}^{rel}}} \right], \end{aligned} \tag{9}$$

⁴⁸Up until this point, we have not discussed the functional form of wages used to calculate the statistics. There is a connection between how wages are used to calculate the statistics (*i.e.* levels or logs) and the form of measurement error assumed. We estimate the statistics using log-wages and hence, we assume that the measurement error in wages follows $w = \eta_w \nu_w$ in levels. Another researcher may prefer to assume measurement error that is additively separable in levels and could then estimate and correct the statistics using wages in levels.

where $\lambda_{\Delta w}^{rel}$ is the reliability statistic for first-differences in wages ($\lambda_{\Delta w}^{rel} = \frac{\sigma_{\eta_{\Delta w}}^2}{\sigma_{\eta_{\Delta w}}^2 + \sigma_{\nu_{\Delta w}}^2} = \frac{\sigma_{\eta_{\Delta w}}^2}{\sigma_{\Delta w}^2}$) and Φ is the CDF of the standard normal distribution.

Thus, we need the reliability ratio for both the level of wages and the first-difference in order to correct the two statistics. The reliability ratio of wages is available from the literature on measurement error in survey data and, given certain assumptions, can also be measured in our data. For example, in the SIPP data we can compare wages across two observations of the same worker-firm match separated by four months. We do not look at adjacent months, because the measurement error is likely to be correlated within the same interview. If we assume that the true wage is constant across the four months within a given job, we can use the correlation of the within-match wages to estimate the reliability ratio in levels⁴⁹

$$\begin{aligned} corr(w_{i,t}, w_{i,t+1}) &= \frac{\sigma_{\eta_w}^2}{\sigma_{\eta_w}^2 + \sigma_{\nu_w}^2} \\ &= \lambda_w^{rel}. \end{aligned}$$

Our estimate using the within-match correlation in the SIPP ($\hat{\lambda}_w^{rel} \approx 0.882$) is quite close to [Bound and Krueger \(1991\)](#) who estimate reliability statistics for men and women by comparing reported income in the CPS to social security records ($\hat{\lambda}_w^{rel} \approx 0.864$). [Bound, Brown, and Mathiowetz \(2001\)](#) review the literature on the correlations between worker and employer earnings and find that correlations in studies using weekly, monthly, and annual earnings are similar (see Section 6.1.2 therein). While SIPP measures monthly earnings and CPS measures weekly earnings, we will use our SIPP estimates of $\hat{\lambda}_w^{rel}$ as [Bound and Krueger \(1991\)](#) do not estimate reliability ratios by education groups, which we need for our results conditional on education.

It is not as straightforward to measure the reliability ratio of the first-difference in wages ($\lambda_{\Delta w}^{rel}$) from survey data alone. [Bound and Krueger \(1991\)](#) estimate reliability statistics for men and women by comparing reported income in the CPS to social security records. We calculate a weighted average of the reliability ratios of wage differences for men and women in CPS data of $\lambda_{\Delta w}^{rel} = 0.711$.⁵⁰ In other words, 71% of the variance in wage differences is signal in CPS data.

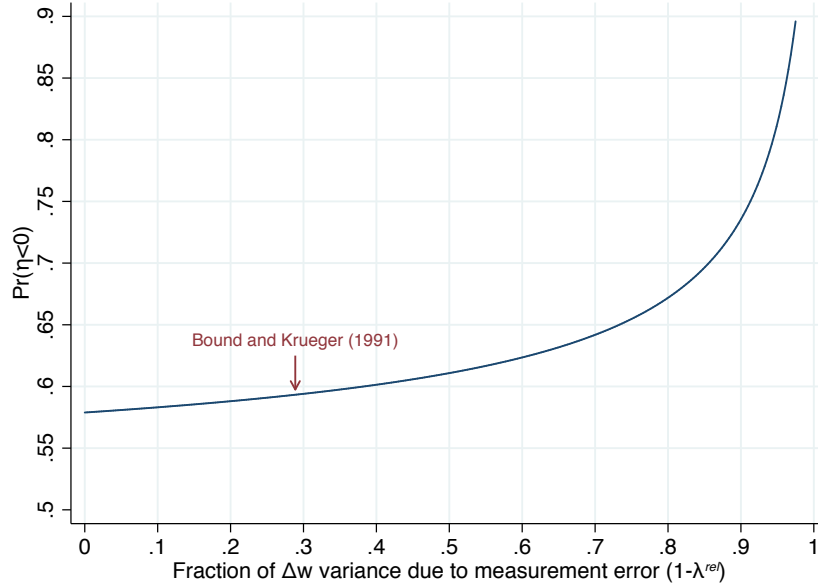
It may not be clear from Equation 9 if measurement error leads to important downward biases for the wage-growth statistic. In Figure 2 we show how, given a measured fraction of 0.576 (our preferred estimate), the true fraction depends on different amounts of measurement error. The x-axis is the fraction of variance due to measurement error, $(1 - \lambda_{\Delta w}^{rel})$. Measurement error would have to be unreasonably large to have a substantive effect on the wage-growth statistic. We use

⁴⁹There are other reasons why wages may change such as bonuses, temporary sickness, inflation coupled with wage rigidities, *etc.* Given the agreement of our reliability estimate using within-job wage changes with [Bound and Krueger \(1991\)](#), who compare reported income to administrative records, we conjecture these other sources do not make an significant contribution to our estimates of the measurement error.

⁵⁰See Table 6 on page 16 in [Bound and Krueger \(1991\)](#), where we use the estimates for classical measurement error for income differences and the gender composition of our baseline sample (57 percent male). While there are differences in the measure the SIPP uses (weekly earnings) with the measure used in [Bound and Krueger \(1991\)](#) (annual earnings), we prefer using their estimates as they directly estimate the measurement error in earnings changes in CPS data.

$\lambda_{\Delta w}^{rel} = 0.711$ from Bound and Krueger (1991) to do the correction and calculate the corrected statistic to be $\hat{Pr}_{corr} = 0.590$.

Figure 2: Correcting the Wage-Growth Statistic for Measurement Error ($\hat{Pr}(\Delta w < 0) = 0.576$)



Notes: This figure shows how the corrected wage-growth statistic depends on different amounts of measurement error assuming the measured statistic of $\hat{Pr}(\Delta w < 0) = 0.576$ from SIPP (see Section 3.4). Using $\lambda_{\Delta w}^{rel} = 0.711$ (Bound and Krueger (1991)), the corrected statistic is $\hat{Pr}_{corr} = 0.590$.

3.4 Results

In this section we present our estimates of the wage-dispersion statistic and wage-growth statistic, with and without the measurement error-corrections presented above.

The results for the wage-dispersion statistics are presented in Table 3. We find that the correlation between the pre- and post earnings are 0.72 and 0.76 in the SIPP and CPS, respectively. Correcting for measurement error increases the correlations to 0.81 and 0.86. Recall, that one minus the correlation is an upper bound on the relative importance of search frictions. This suggests that search frictions play a small role in generating wage dispersion compared to worker differences. Looking at educational sub-groups decreases the correlation some. This is only natural, since the variance of wages is lower in the sub-samples (*i.e.* we now only look at within group variation). The

⁵¹About 9 percent of the CPS-DWS sample reports identical pre- and post-displacement nominal wages. If these are not included, then the estimated uncorrected wage-dispersion statistics are 0.685, 0.606, 0.650, 0.638, for the full sample, high school, some college, and college graduate samples, respectively.

Table 3: Results on Wage-Dispersion Statistic

	N	$corr(w^{pre}, w^{post})$	$\frac{corr(w^{pre}, w^{post})}{\lambda_w^{rel}}$
Displaced (SIPP)	1914	0.755 (0.012)	0.855 (0.014)
High School or less	795	0.658 (0.024)	0.812 (0.030)
Some College	686	0.658 (0.026)	0.786 (0.031)
College Graduate	433	0.724 (0.029)	0.826 (0.033)
Plant closure (CPS-DWS)	2241	0.715 (0.012)	0.810 (0.014)
High School or less	813	0.632 (0.023)	0.780 (0.029)
Some College	916	0.681 (0.019)	0.813 (0.023)
College Graduate	512	0.673 (0.034)	0.768 (0.039)

Notes: Standard errors are in parenthesis. Sample selection: prime aged (25-54 years old), full-time, private-sector workers, not working in agriculture or construction, who made a full-time private sector to full-time private sector transition. Displaced workers in the SIPP sample includes workers who were displaced due to a layoff, employer bankruptcy, or slack work conditions. The CPS-DWS sample includes workers who were displaced due to a plant closing.⁵¹ λ_w^{rel} is the reliability ratio for measurement error in wages. Education-specific reliability ratios are calculated using the SIPP cross-sectional dataset. We estimate λ_w^{rel} to be 0.883, 0.811, 0.837, and 0.877 for the full sample, high school or less, some college, and college graduates, respectively. See Section 3.3 for more details. Standard errors are estimated via 10,000 bootstrap samples. *Sources: SIPP: Survey of Income and Program Participation 1996, 2001, 2003, and 2008 panels. CPS-DWS Current Population Survey - Displaced Workers Survey 1984-2016.*

overall result is that search frictions explain about 15% to 25% of the variance in wages depending on the subgroup.

The results for the wage-growth statistics are presented in Table 4. We measure the fraction $\Pr(\Delta w < 0) = 0.576$ using the SIPP data and $\Pr(\Delta w < 0) = 0.587$ using the CPS data. Correcting the fraction for measurement error increases it by about 0.015 in both samples to $\tilde{Pr}(\Delta w < 0) = 0.590$ and $\tilde{Pr}(\Delta w < 0) = 0.603$, respectively for SIPP and CPS. We use the corrected statistics to calculate the bounds on receiving no job offers while employed (see Equation 4). The results suggest that at least about 40% workers experience no frictional wage growth over an employment spell in the sense that they receive additional offers. Finally, we report the measurement error corrected implied value of κ , which is denoted by $\hat{\kappa}_{corr}$. Our results imply that, on average, workers are more likely to receive a job destruction shock than a job offer shock when employed. The results for the education sub-samples reveal important heterogeneity. On one hand, workers with less than a college degree exhibit some frictional wage growth, where the fraction of workers with no frictional wage growth is smaller and $\hat{\kappa}_{corr}$ is above one. On the other hand, our results indicate that workers with a college degree experience almost no frictional wage growth. We find that at least 60 percent to 100 percent of college-educated workers experience no frictional wage growth during an employment spell, in the SIPP and CPS respectively. Put another way, we estimate $\hat{\kappa}_{corr}$ to be about 0.5 and zero for workers with a college degree, in the SIPP and CPS respectively.

Finally, we investigate how the statistics vary over time. Panel A in Figure 3 shows the wage-dispersion statistic for both the CPS and SIPP across time uncorrected for measurement error. While there is no clear pattern with respect to the business cycle, there is a positive trend. This is evidence that frictional wage dispersion has been decreasing over time since the 1980s. For example, we estimate frictional wage dispersion to account for about 25 percent of the variance in wages in the 1980s (1982-1990), while only about 14 percent in the last ten years (2006-2014) of the CPS-DWS. Panel B in Figure 3 shows the wage-growth statistic for both samples. While the estimates are not very precise, there is a clear counter-cyclical relationship with a higher fraction of workers earning lower wages if they lose their job during (or just after) a recession. This is consistent with our analysis, where we would expect that the average number of job offers accumulated by workers is at its highest at the end of an expansion.

In this section, we computed the wage-growth and wage-dispersion statistics using two different samples. There are two overall conclusions based on these findings. First, the estimated wage-dispersion statistics show that search frictions explain about 20 percent of wage dispersion over the entire time period, but has become less important recently (about 14 percent). Second, we do not see evidence of a lot of frictional wage growth. We estimate that between 40 and 80 percent of

⁵²About 9 percent of the CPS-DWS sample reports identical pre- and post-displacement nominal wages. These are all recorded as wage losses in real terms. If these are not included, then the estimated uncorrected wage-growth statistics are 0.534, 0.571, 0.556, 0.432, for the full sample, high school, some college, and college graduate samples, respectively.

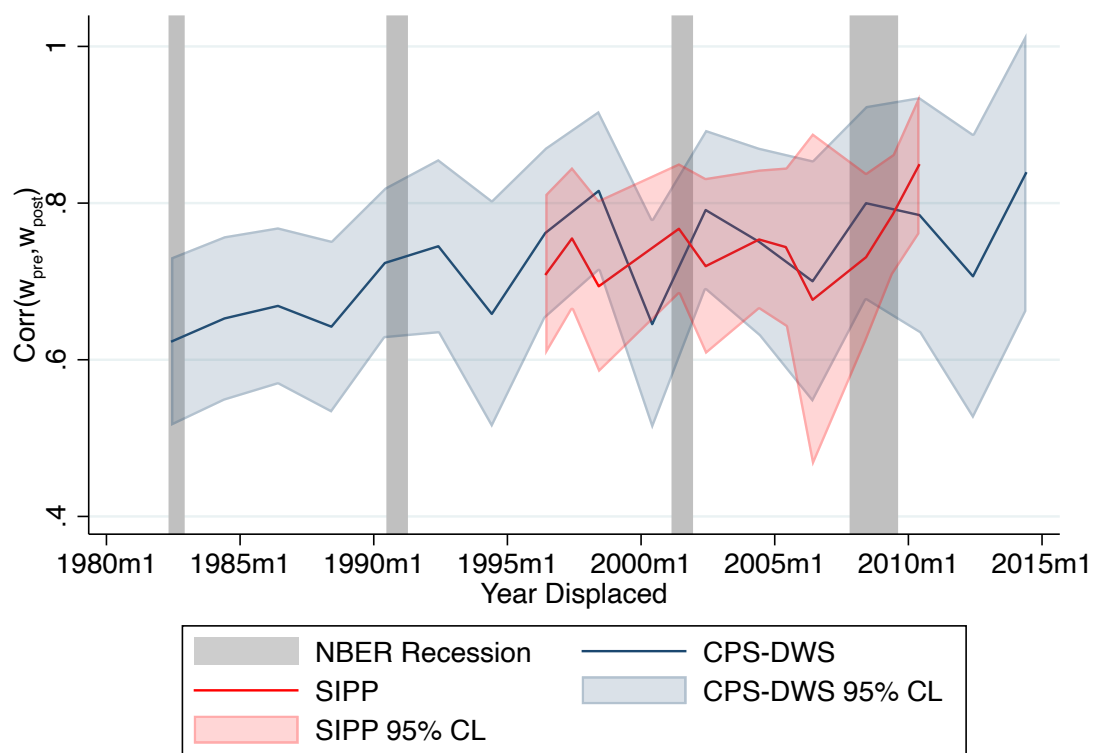
Table 4: Results on Wage-Growth Statistic

	N	$\hat{Pr}(\Delta w < 0)$	$\hat{Pr}_{corr}(\Delta w < 0)$	$\hat{Pr}_{corr}(n = 1)$	$\hat{\kappa}_{corr}$
Displaced (SIPP)	1914	0.587 (0.011)	0.603 (0.013)	[0.381, 0.794] (0.079, 0.026)	0.872 (0.154)
High School or less	795	0.556 (0.018)	0.566 (0.021)	[0.602, 0.867] (0.124, 0.041)	0.492 (0.192)
Some College	686	0.643 (0.018)	0.668 (0.021)	[0.0, 0.664] (0.125, 0.042)	1.841 (0.407)
College Graduate	433	0.557 (0.024)	0.567 (0.028)	[0.598, 0.866] (0.171, 0.057)	0.498 (0.267)
Plant closure (CPS-DWS)	2241	0.576 (0.010)	0.590 (0.012)	[0.460, 0.820] (0.074, 0.025)	0.725 (0.132)
High School or less	813	0.605 (0.017)	0.624 (0.020)	[0.255, 0.751] (0.120, 0.040)	1.136 (0.275)
Some College	916	0.597 (0.016)	0.615 (0.019)	[0.311, 0.770] (0.113, 0.038)	1.013 (0.242)
College Graduate	512	0.492 (0.022)	0.491 (0.026)	[1.000, 1.000] (0.072, 0.024)	0.000 (0.082)

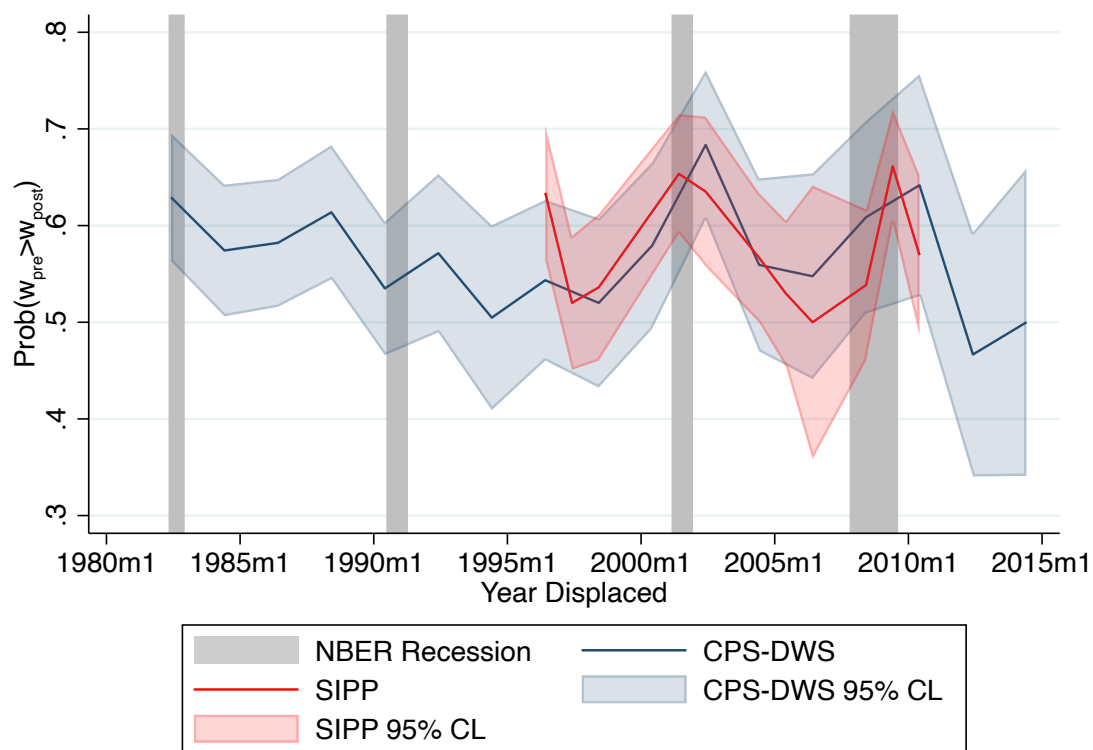
Notes: Standard errors in parenthesis. Sample selection: prime aged (25-54 years old), full-time, private-sector workers, not working in agriculture or construction, who made a full-time private sector to full-time private sector transition. Displaced workers in the SIPP sample includes workers who were displaced due to a layoff, employer bankruptcy, or slack work conditions. The CPS-DWS sample includes workers who were displaced due to a plant closing.⁵² $\lambda_{\Delta w}^{rel}$ is the average reliability ratio for men and women from Bound and Krueger (1991) ($\lambda_{\Delta w}^{rel} = 0.711$). $\hat{Pr}_{corr}(n = 1)$ shows the bounds on the fraction of workers receiving zero job offers during the last employment spell after correcting for measurement error. $\hat{\kappa}_{corr}$ is the implied κ after correcting for measurement error. See Section 3.3 for more details. Standard errors are estimated via 10,000 bootstrap samples. Sources: SIPP: Survey of Income and Program Participation 1996, 2001, 2003, and 2008 panels. CPS-DWS Current Population Survey - Displaced Workers Survey 1984-2016.

Figure 3: Time Series of Sufficient Statistics

(a) Wage-Dispersion Statistic



(b) Wage-Growth Statistic



workers no frictional wage growth during an employment spell. We do find however, that frictional wage growth becomes more important at the end of economic expansions. The job offer and job destruction rates implied by the wage-growth statistic are different than those implied by looking at job-to-job and job-to-unemployment transition rates in the data.⁵³ In the next section, we structurally estimate two versions of a search model in order to show how the wage-growth statistic can be reconciled with observed the transition rates within a search model.

4 Using the Statistics in Structural Estimation

In this section we estimate two on-the-job search models with human capital accumulation. The purpose of this is three-fold. First, we show how search models can be consistent with both our frictional wage-growth statistic and the observed job-to-job mobility in the data. At least two mechanisms can bridge the gap: involuntary job-to-job transitions and compensating differentials. Second, we demonstrate how our two proposed statistics can be used to estimate on-the-job search models. Third, we can use the estimated models to decompose cross-sectional wage dispersion and life-cycle wage growth into frictional and learning-by-doing human capital components. Although the fraction of displaced workers experiencing a wage loss identifies κ , a model is needed to *quantify* the relative importance of the mechanisms driving life-cycle wage growth.

4.1 Model

We set up a continuous-time, infinite-horizon model of the labor market, where agents discount the future at rate ρ . Workers have *ex ante* heterogeneous levels of permanent ability (α). Furthermore, they accumulate human capital (k) via learning-by-doing when employed, where k is discrete and has finite support ($k \in \{0, \dots, K\}$). Workers enter the labor market with $k = 0$ and draw their permanent ability from the ability distribution $H(\alpha)$. Firms are heterogeneous in their productivity (p) and in the non-pecuniary aspect of the job (z). They produce log output $\alpha + p + f(k)$. Workers draw job offers from the bivariate offer distribution $F(p, z)$.

Unemployed workers receive a flow utility that is a function of their ability and learning-by-doing human capital ($u_0(\alpha, k)$). While unemployed, they receive job offers at rate λ^u and do not accumulate human capital. Once they receive a job offer they must choose either to accept it and become employed or reject it and remain unemployed.⁵⁴

⁵³ The observed JJ flow rate is the offer arrival rate times the fraction of jobs accepted, which in the job ladder model is

$$\Pr(J \rightarrow J) = \lambda^e \int [1 - F(w)] dG(w) = \frac{\delta(\lambda^e + \delta) \ln(\frac{\lambda^e + \delta}{\delta}) - \lambda^e \delta}{\lambda^e}.$$

This can be derived analogously to Equation 6 in our paper or using steady state arguments as in is [Hornstein, Krusell, and Violante \(2011\)](#) (Equation 15 in their paper). In Section 4.2, we estimate the annual transition rates $\Pr(J \rightarrow U) = \delta = 0.109$ and $\Pr(J \rightarrow J) = 0.109$. The implied λ^e from the above equation is then $\lambda^e = .427$, which corresponds to $\kappa = 3.922$, which is more than 4.5 times higher than the $\hat{\kappa}_{corr} = 0.73 - 0.87$ we estimate from the wage-growth statistic.

⁵⁴In our empirical specification we assume that unemployed workers choose to accept all job offers.

An employed worker with ability α and human capital k receives flow utility $u_1(\alpha, p, k, z)$ when working at a (p, z) firm.⁵⁵ Employed workers receive job offers at rate λ^e , which they can accept or decline. They also receive involuntary job offers at rate λ^d , which can only be rejected by entering unemployment.⁵⁶ At rate λ_h , their human capital increases by one unit via learning-by-doing for $k < K$. Finally, they receive job separation shocks at rate δ , and become unemployed.⁵⁷

4.2 Estimation

In this section we describe the parametrization and estimation of the model.

Model Specification and Parameterization The flow utility of an employed worker is additively separable in $\log(\text{wage})$ and the non-pecuniary aspect,

$$u_1(\alpha, p, k) = \log(\text{wage}) + z = \alpha + f(k) + p + z.$$

Worker ability is normally distributed following $\alpha \sim N(\mu, \sigma_\alpha)$. We assume that p and z are independently distributed.⁵⁸ We let p and z be log-normally distributed, where $p \sim LN(0, \sigma_p)$ and $z \sim LN(0, \sigma_z)$.⁵⁹ We let $f(k) = \beta_1 k + \beta_2 k^2$ and set $K = 20$. Finally, wages are observed with measurement error $\epsilon \sim N(0, \sigma_\epsilon)$.

For reasons of tractability, we assume that unemployed workers accept all job offers.⁶⁰ Thus, we implicitly assume that the value of staying unemployed and taking a job at the worst firm is the same for all k .⁶¹

Poisson rates are measured in annual terms. We fix $\rho = 0.05$ and normalize $\lambda_h = 1$, which leaves eleven free parameters $(\delta, \lambda^u, \lambda^e, \lambda^d, \mu, \sigma_p, \sigma_\alpha, \chi_1, \chi_2, \sigma_\epsilon, \sigma_z)$.

We estimate two versions of the model to show how our statistics discipline wage dispersion and wage growth in different settings. In the first version, the "wage-ladder" model, we fix $\sigma_z = 0$

⁵⁵Note, that workers simply receive their full productivity, $\alpha + p + f(k)$. Thus, firms take a passive role in this model. In Section 4, we will explore how sequential bargaining affects the wage-growth statistic.

⁵⁶Again, note that in our specification unemployed workers will choose to accept all job offers, so employed workers will always accept involuntary job offers over unemployment.

⁵⁷See Appendix Section D.1 for the value functions.

⁵⁸Taber and Vejlin (2020) find that this is approximately the case in a model without any restrictions on the covariance between p and z .

⁵⁹In a previous version, we have also estimated the model with exponential distributions. This choice is not critical, but using log-normal distributions, we can more easily correlate the two distributions, which is done as a robustness check.

⁶⁰Since entering employment carries the option value of increasing human capital the flow utility of an unemployed worker will be higher than or equal to that of working for the firm which offers the lowest utility. However, calculating exactly how the flow utility depends on the level of human capital such that this is the case complicates the solution.

⁶¹We are not the first to assume this. Our model is closely related to that of Bagger, Fontaine, Postel-Vinay, and Robin (2014), which introduce human capital into a sequential-auction model with bargaining. In order to avoid dealing with reservation wages they implicitly make the same assumption.

and allow for involuntary job offers. In the second version, the "compensating differential" model, we fix $\lambda^d = 0$ and include non-pecuniary aspects of the job. Each model has ten free parameters.

All workers enter the model as unemployed and with zero human capital ($k = 0$). We simulate workers for 25 years and auxiliary parameters are calculated using data from year 2 to year 25.⁶² As the solution and simulation of the model are not a novel aspect of this paper, we provide a full description in Appendix Section D.2.

Auxiliary Parameters We estimate the model by indirect inference (Gourieroux, Monfort, and Renault 1993) using the bootstrap covariance matrix for the auxiliary parameters as a weighting matrix. We use two datasets to measure the auxiliary parameters. The 1979 National Longitudinal Survey of Youth (NLSY79) is a nationally representative panel that follows respondents starting at ages 14-22 when first interviewed in 1979. NLSY79 respondents have been interviewed either annually or biennially since 1979. We calculate transition rates and wage statistics using the NLSY79 and use the SIPP (see Section 3.1) to calculate the wage-growth and wage-dispersion statistics and measurement error moments.⁶³ All auxiliary parameters are calculating for workers who hold full-time private-sector employment. Here we give a brief description of the auxiliary parameters used for estimation.⁶⁴

The auxiliary parameters targeting the transitions parameters are calculated as the yearly probabilities of being displaced ($\Pr(E \rightarrow U)$), taking a job out of unemployment ($\Pr(U \rightarrow E)$), and making a job-to-job transition ($\Pr(E \rightarrow E)$) in the NLSY79. These transition rates help identify the job offer and job destruction rates in the models ($\lambda^u, \lambda^e, \delta$).

Frictional wage dispersion and wage growth are disciplined by the statistics studied in this paper. We use the SIPP to calculate the frictional wage dispersion ($\text{corr}(w_x^{\text{pre}}, w_x^{\text{post}})$) and the frictional wage-growth statistics ($\Pr(w_x^{\text{post}} < w_x^{\text{pre}})$). These auxiliary parameters discipline the amount of frictional wage dispersion and frictional wage growth in the model. The wage-dispersion statistics identifies the variation in firm types, σ_p . In the wage-ladder model, the wage-growth statistic helps identify the involuntary job offer arrival rate (λ^d). While in the compensating differentials model, the wage-growth statistic helps identify the non-pecuniary parameter (σ_z). The auxiliary parameter for the measurement error, σ_ϵ is the within-job wage correlation ($\text{Corr}(w_t, w_{t+0.33} | \text{within match})$). It uses wages measured from two consecutive waves (four months apart) in the CS SIPP sample (see Section 3.1).

Lifecycle wage growth parameters are measured by estimating a Mincer regression on experience and experience squared in the NLSY79. The intercept (ζ_0), linear coefficient (ζ_1), and quadratic

⁶²Ignoring the first years after labor market entry is also done in Bagger, Fontaine, Postel-Vinay, and Robin (2014) in order to avoid noise in the real data from the early part of a labor market, where the transition from school to work can take some time and this is not modelled.

⁶³Notice that the sample selection for the structural estimation is different than in Section 3.1 and the values differ slightly for that reason. The main difference between the two calculations is that the model estimation conditions on 2-25 years of experience.

⁶⁴The full details on how the auxiliary parameters are constructed can be found in Appendix Section E.

coefficient (ζ_2) are used as auxiliary parameters to identify μ, χ_1, χ_2 , respectively. The standard deviation (σ_w) of wages are also measured in the NLSY79 and helps to identify σ_α .

Fit and Estimates Table 5 shows the fit of the auxiliary parameters for each model.

Table 5: Auxiliary Parameter Fit

	Wage Ladder Model	Compensating Differential Model	Data
$\Pr(J \rightarrow U)$	0.109	0.109	0.109
$\Pr(U \rightarrow J)$	0.639	0.639	0.639
$\Pr(J \rightarrow J)$	0.109	0.109	0.109
$\Pr(w_x^{post} < w_x^{pre})$	0.570	0.570	0.570
σ_w	0.574	0.574	0.574
$corr(w_x^{pre}, w_x^{post})$	0.755	0.755	0.755
ζ_0	2.292	2.290	2.291
ζ_1	0.028	0.028	0.028
$\zeta_2 \times 100$	-0.033	-0.034	-0.034
$Corr(w_t, w_{t+0.33} \text{within match})$	0.882	0.882	0.882

The model is simulated using 5,000,000 worker histories from year 0 to year 25. Notice that the data values differ slightly from Section 3.1. The main reason between the two calculations is that the model estimation conditions on 2-25 years of experience.

Both estimated models fit the data to almost four significant digits. This is not surprising given that we have just as many structural parameters as auxiliary parameters, but it is not guaranteed in non-linear models.

In Table 6 we show the resulting parameter estimates. Almost all are statistically significant.⁶⁵

We find that the variance of worker ability is much higher than the variance of firm productivity and measurement error. This is expected given the high correlation of pre- and post-displacement wages, which we show in Section 2.2 is closely related to the fraction of the total variance explained by between worker differences. Also notice, that we find λ^d to 0.052 in the wage-ladder model, implying that workers receive an involuntary job offer shock on average once every seventeen years. It is the relatively high value of λ^d that allows us to reconcile the large job-to-job flows with the wage-growth statistic.⁶⁶ On average an involuntary job-to-job shock throws the worker down

⁶⁵Standard errors are computed using the formula in [Gourieroux, Monfort, and Renault \(1993\)](#). Because of simulation error the auxiliary parameters are not continuous in the parameters. Thus, taking numerical derivatives cannot be done in the usual way. In order to overcome this problem we do a Taylor expansion around the estimated parameter values. Specifically, we simulate the auxiliary parameters for different values of the parameters using a uniform grid of parameter values around the estimated parameter (25 percent deviation) and then fit the binding function by a linear function in the parameter values. We then use this function to take the derivative at the estimated parameter values. The numerical derivatives are robust to using upto a third order polynomial and using different values of the deviations.

⁶⁶The estimate is fairly close to [Bagger and Lentz \(2019\)](#), who finds a value of 0.078 using Danish data.

Table 6: Structural Parameter Estimates

	Wage-Ladder Model		Compensating Differential Model	
	Parameter Value	Std. Err.	Parameter Value	Std. Err.
δ	0.116	0.002	0.116	0.002
λ^u	1.108	0.029	1.109	0.029
λ^e	0.173	0.034	0.424	0.010
λ^d	0.052	0.010	-	-
σ_α	0.476	0.007	0.475	0.007
σ_p	0.194	0.010	0.189	0.009
μ	1.284	0.010	1.279	0.010
χ_1	0.018	0.003	0.018	0.003
$\chi_2 \times 100$	0.017	0.011	0.015	0.012
σ_ϵ	0.196	0.003	0.220	0.003
σ_z	-	-	0.312	0.064

Standard errors are computed using the formula in [Gourieroux, Monfort, and Renault \(1993\)](#). The standard deviation of the firm effect (p) is 0.200 and 0.194 in the wage-ladder model and the compensating differential model, respectively. The standard deviation of the non-pecuniary aspect of a job (z) is 0.336 in the compensating differential model.

the job ladder. This makes future job-to-job transitions more likely, since the worker is now at a lower rung in the firm productivity distribution and thereby accepts more of the offers that he receives.

In the compensating differential model, we find $\sigma_z = 0.312$, which implies that the standard deviation of the non-pecuniary aspect is 0.336. Thus, the non-pecuniary aspect is more important than the differences in firm productivity ($\sigma_p = 0.189$). Because the non-pecuniary aspect of job is so important, workers often select jobs primarily based on that. The compensating differential model matches the large job-to-job flows in the data by estimating a high λ^e , but does so without much frictional wage growth, consistent with the wage-growth statistic.

If we calculate κ from the wage-ladder and compensating differential model, we get 1.03 and 3.66, respectively. The inferred κ from the wage-growth statistic is 0.872. The small difference between the implied κ and the κ in the wage-ladder model is most likely due to differences in the sample compositions. κ in the compensating differential model is not consistent with the wage-growth statistics calculation. This is not unexpected either, since in the compensating differential model the wage is not an order statistic of the job value. Regardless, we will see that the inference on wage growth is consistent across both models.

Non-targeted Auxiliary Parameters In Table 7 we present non-targeted auxiliary statistics that are not used in the estimation procedure next to values from our data.

⁶⁷The most common "involuntary" reasons are "On layoff", "Other family/personal obligations", "Discharged/fired", "Employer bankrupt", "Employer sold business", "Job was temporary and ended", and "Slack work or business conditions".

Table 7: Non-targeted Auxiliary Parameters

	Wage-Ladder Model	Compensating Differential Model	Data	Std. Err.
$E(\Delta w JJ)$	0.088	0.087	0.076	(0.007)
$Var(\Delta w JJ)$	0.147	0.146	0.160	(0.007)
Fraction JJ Wage Loss	0.396	0.411	0.368	(0.009)
$E(\Delta w JUJ)$	-0.069	-0.071	-0.068	(0.010)
$E(\Delta w JUJ, high-tenure)$	-0.092	-0.091	-0.104	(0.018)
Fraction Invol JJ	0.443	0.000	0.216	(0.004)

The table reports non-targeted auxiliary statistics, which are commonly used in the estimation of search models. JUJ moments are calculated using the SIPP JUJ sample, and the JJ moments use the CS SIPP sample. "Voluntary" JJ transitions are defined as transitions where workers change jobs for the following three reasons (1) "Quit to take another job (2) Unsatisfactory working arrangements", and (3) Quit for some other reason. The remaining reasons are unrelated to the jobs and are considered involuntary.⁶⁷ High-tenure is defined as having more than 3 years of tenure. Column (4) shows the standard error on the estimate from the data.

The non-targeted statistics in our estimated models are not as far away from the data as we would have expected. However, there are still differences. The average wage change for workers making a job-to-job transition is 7.6 percent in the data, but it is 16 percent higher in the models.

Turning to the fraction of workers who experience a JJ wage loss, we find the moment to be around 10 percent higher than in the data. Again, if we had targeted this moment instead of the wage-growth statistics then we would have needed a lower λ_d again implying that frictional wage growth would be higher.

The two next rows report the wage change around unemployment. In average wage change for the full population is the same in the data as in the models, but the wage change is larger in the data for high-tenure workers. In general, these two moments would speak to the identification of σ_p . If we had wanted to target the wage change for high-tenure workers instead of the wage-dispersion statistic then we would have needed a higher estimate of σ_p , which would imply that frictional wage dispersion would have been higher.

Finally, in the last row, we report the fraction of job-to-job transitions that are voluntary. The large difference between the fraction of involuntary JJ transitions in the wage-ladder model and the data is due to the fact that the only way the model can generate a low enough wage-growth statistic, while still matching the observed transition rates, is by setting λ^d fairly high resulting in many involuntary job transitions. We will return to this problem in a robustness exercise, where we estimate a joint version of the model.

4.3 Results

We use the model to make two kinds of decompositions. First, we show how much of the variance of wages is caused by frictional wage dispersion and secondly, how much of life-cycle wage growth is caused by frictional wage growth compared to human capital induced wage growth.

Frictional Wage Dispersion In order to show the importance of frictional wage dispersion, we decompose the variance of wages in Table 8 for the wage-ladder model and the compensating differential model.⁶⁸ The first thing to note, is that even though the two models substantially differ, the estimated variance decompositions are very similar. First, we take out variance caused by measurement error, since this is not relevant for workers. In both models we estimate that around 12 percent of the variance in wages is due to measurement error. This decreases the variance from 0.331 to 0.293 in the two models. We then decompose the true wage variance into four components

$$var(\tilde{w}) = var(\alpha) + var(p) + var(f(k)) + 2cov(p, f(k))$$

where \tilde{w} is the true wage without measurement error. The co-variances between the worker type, α , and the firm component, p , and the human capital component, $f(k)$, are zero by construction.

Table 8: Wage Variance Decomposition

	Wage-Ladder Model		Compensating Differential Model	
	Variance	Share	Variance	Share
Wage Var	0.331		0.331	
Var(ϵ)	0.039		0.039	
Wage Var Without Meas Err (ϵ)	0.293		0.293	
Var(α)	0.226	0.774	0.225	0.770
Var(p)	0.043	0.146	0.044	0.150
Var($f(k)$)	0.019	0.064	0.019	0.066
2 Cov($p, f(k)$)	0.005	0.016	0.004	0.015

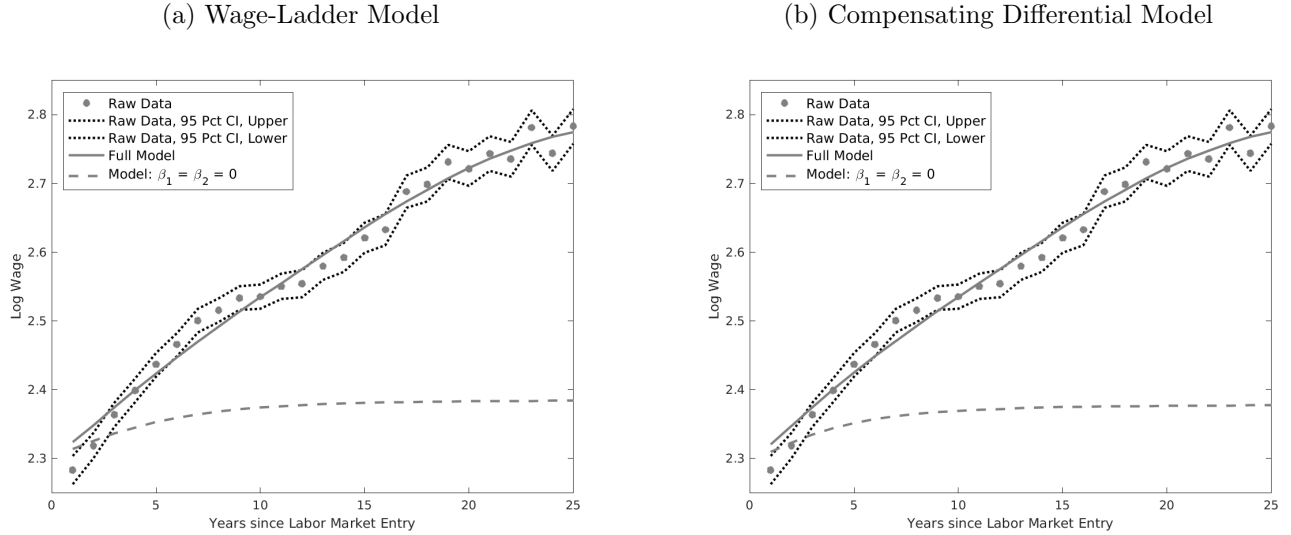
Notes: The table shows a linear decomposition of the wage variance taking out measurement error for each model. It decomposes the true variance into a worker component (Var(α)), a firm component (Var(p)), a human capital component (Var($f(k)$)) and the covariance between the worker and firm components (2 Cov($p, f(k)$)).

From the results it is clear that the main part of the variance is explained by worker heterogeneity as also suggested by the wage-dispersion statistic, which showed that the correlation of pre- and post-displacement wages were 0.755. Differences in firm productivity account for about 15 percent across the two models, while human capital accumulation matters to a smaller extent, about 6-7 percent. This is to be expected as a simple Mincer regression of wages on experience typically do not explain much more than 5-7 percent of wage variation.

⁶⁸This is the variance of wages measured in the annual repeated cross-sections from year 1 to year 25 in the labor market.

Life-Cycle Wage Growth We now move to the second question of how much of life-cycle wage growth is caused by frictional wage growth. Figure 4 contains two sub-figures, one for each model. In each sub-figure we plot three graphs. First, we plot the average wage by years since labor market entry from the real data together with the 95 percent confidence interval. Second, we plot the average wage from the model and finally, we plot the average wage from the model imposing that there is no human capital accumulation ($\chi_1 = \chi_2 = 0$), so the only reason wages go up over the life-cycle is frictional wage growth due to workers climbing the productivity ladder.

Figure 4: Life-Cycle Wages



Notes: The plot of the real data ("Raw Data") uses the same NLSY data as we used in the estimation except that here we also plot data after the first year in the labor market. The simulated data is from the model, where we simulate workers from labor market entry (at time zero), where they enter as unemployed, to 25 years after. Frictional wage growth is measured by simulating the model setting $\chi_1 = \chi_2 = 0$, while human capital wage growth is the difference in growth between total wage growth and frictional wage growth.

Figure 4 shows that the model fit is excellent for both models in terms of matching the life-cycle wage profile from the data. As was the case for the variance decomposition of wages, the results from the two models are strikingly similar. Comparing the full models with the models without any return to human capital accumulation, it is clear that early in the life cycle both human capital accumulation and frictional wage growth play important roles for the overall wage growth. However, after the first 5-10 years frictional wage growth is negligible.

Table 4 decomposes total wage growth from labor market entry to 25 years into human capital wage growth and frictional wage growth. Search frictions account for about 15 percent of the total wage growth over the first 25 years of the life cycle in both models.

Finally, as indicated by Figure 4 the relative roles of human capital and frictions in generating wage growth varies over the life-cycle. In Figure 5 we show the differences in yearly average wages

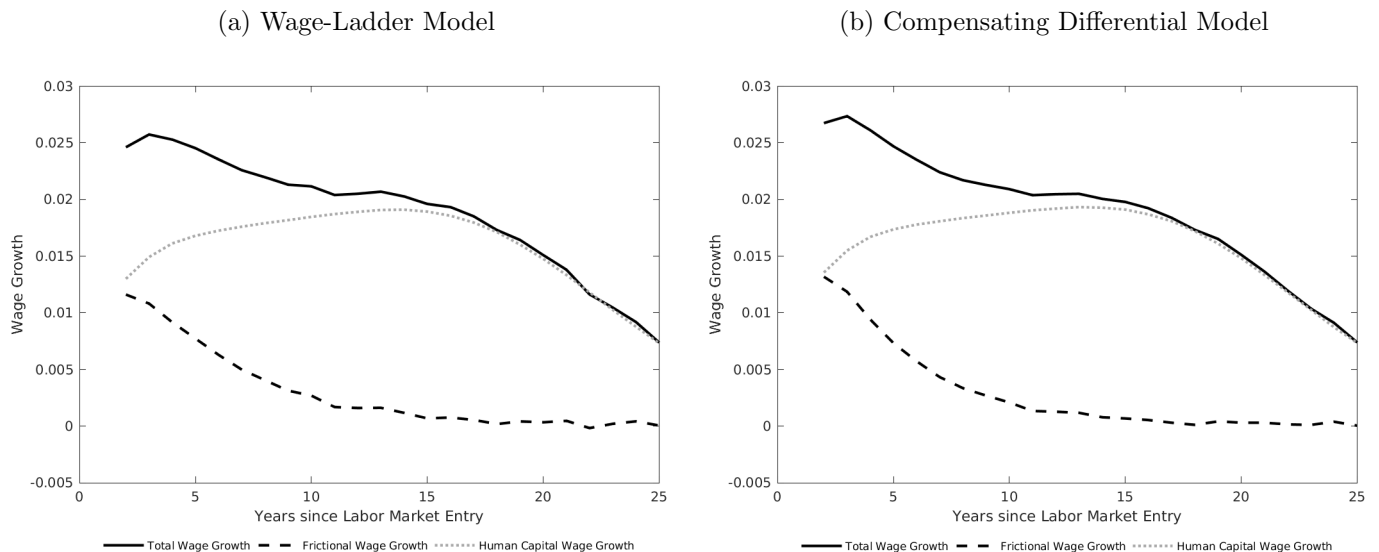
Table 9: Decomposition of Wage Growth At 25 Years of Experience

	Wage-Ladder Model		Compensating Differential Model	
	Wage Growth	Share	Wage Growth	Share
Total Wage Growth	0.451		0.455	
Human Capital Wage Growth	0.381	0.844	0.387	0.851
Frictional Wage Growth	0.070	0.156	0.068	0.149

Notes: The table decompose total wage growth from labor market entry to 25 years after into human capital wage growth and frictional wage growth. Frictional wage growth is measured by simulating the model setting $\chi_1 = \chi_2 = 0$, while human capital wage growth is the difference in growth between total wage growth and frictional wage growth.

in the full model (labeled Total Wage Growth) and in the model without any human capital accumulation (labeled Frictional Wage Growth). Finally, we show the difference between the two (labeled Human Capital Wage Growth).⁶⁹

Figure 5: Wage Growth Decomposition



It is clear from Figure 5 that in the early part of a worker's career frictional induced wage growth is actually around the same level as human capital wage growth. After about 10 years, the average frictional wage growth is zero.

We draw the following conclusions based on the results presented from the structural model.

⁶⁹Notice that the reason that human capital wage growth has an inverse U-shape is that the return to human capital is convex ($\chi_2 > 0$), which causes the small increase in the growth rate until around year 15. Human capital wage growth begins to decline as workers start to obtain the maximum human capital level, K . Recall, that learning-by-doing human capital arrives stochastically.

First, the different κ 's implied by the two models suggest that one should be careful by estimating involuntary job offer shocks from transition rates, since a model with compensating differentials would imply a very different value. Second, even though the κ 's were very different, the amount of frictional wage growth in the two models were surprisingly similar. This shows how the wage-growth statistic can discipline the frictional wage growth across different types of models.

4.4 Robustness of Structural Estimation

We perform several robustness checks of the estimated models. First, we estimate versions of the compensating differential model, where firm productivity and non-pecuniary aspects are allowed to be correlated with correlations of -0.17 and -0.5 (Models (1) and (2)). Second, we estimate a version of the model, where the job offer distributions of employed and unemployed differ, which departs from the assumptions needed for the derivation of the statistics (Model (3)). Finally, we estimate a version of the model, which encompass both of the baseline models (Models (4)). We discuss each model and the results of the estimations in online Appendix Section F.3. We conclude that our estimates of frictional wage growth and frictional wage dispersion are robust to violations of the identifying assumptions examined here.⁷⁰

5 Conclusions

In this paper, we present two sufficient statistics. The first is the correlation of pre- and post-displacement wages, which is informative of frictional wage dispersion. The second is the fraction of workers earning less after an unemployment spell, which is informative of frictional wage growth. We show how these statistics are informative across a large class of search models independent of wage offer distributions and other labor market parameters. We estimate the statistics using displaced workers in both CPS and SIPP data. We find that frictional wage dispersion explains less than 20 percent of the variation in wages and likewise, workers receive few wage improving offers during an employment spell. In other words, the wage-growth statistic implies modest frictional wage growth. While there is not much heterogeneity in the wage-dispersion statistic, we find that college-educated workers experience almost no frictional wage growth. Calculating the two statistics over time, we find that frictional wage dispersion has declined by about half since the 80's. This is in line with improvements in the matching technology such as the emergence of online job databases. Furthermore, frictional wage growth is higher at the end of expansion periods, which is consistent with workers receiving more jobs offers during periods of economic growth. We use the statistics to estimate two different models of the labor market: a wage-ladder model and a compensating differential model. While the implied job offer rates depend on the details of the model, the importance of frictional wage dispersion and frictional wage growth is estimated to be

⁷⁰See identifying assumptions in Section 2.

almost identical across the two models. Frictional wage dispersion is around 16-17 percent of total wage dispersion while frictional wage growth is about 15 percent of the life-cycle wage growth.

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A Mathematical Derivations

A.1 Derivation of Frictional Wage-Dispersion Statistic: $Cov(w^{post}, w^{pre})$

Let $\mu_{xs} = E_{F_{xs}}[w_{xs}^{post}]$ be the conditional mean of the job offer distribution for workers of type x in economic state s and $\Delta\mu_{xs} = E_{G_{xs}}[w_{xs}^{pre}] - \mu_{xs}$ be the difference in the conditional

means of the $G_{xs}(w)$ and the $F_{xs}(w)$ distributions for workers of type x in economics state s . If the post-displacement wage w_{xs}^{post} is independent of the pre-displacement wage w_{xs}^{pre} , then the population covariance is

$$Cov(w^{post}, w^{pre}) = Var(\mu) + Cov(\mu, \Delta\mu).$$

The covariance depends only on the variation in the means of the worker type and economic state specific distributions and is independent of the shape of the distributions.

There are three distributions that need to be integrated over to calculate the covariance: (1) the expectation over types and states denoted as E_{xs} , (2) the expectation over the wage offer distribution conditional on type x and state s ($E_{F_{xs}}$), and (3) the expectation over the pre-displacement wage distribution conditional on type x and state s ($E_{G_{xs}}$). Without loss of generality, we decompose wages into the type- and state-specific mean μ_{xs} and the demeaned wage.

$$\begin{aligned} w_{xs}^{post} &= \mu_{xs} + \varepsilon_{xs} \\ w_{xs}^{pre} &= \mu_{xs} + \eta_{xs}. \end{aligned}$$

Note that, by definition of μ_{xs} , $E_{F_{xs}}[\varepsilon_{xs}] = 0$, while $E_{G_{xs}}[\eta_{xs}] = \Delta\mu_{xs}$ has a non-zero mean.

The covariance is

$$cov(w^{post}, w^{pre}) = E[w^{post}w^{pre}] - E[w^{post}]E[w^{pre}]. \quad (10)$$

The first term on the RHS in Equation 10 becomes

$$\begin{aligned} E[w^{post}w^{pre}] &= E_{xs} \{ E_{F_{xs}G_{xs}}[w_{xs}^{post}w_{xs}^{pre}] \} \\ &= E_{xs} \{ E_{F_{xs}G_{xs}}[\mu_{xs}^2 + \varepsilon_{xs}\mu_{xs} + \mu_{xs}\eta_{xs} + \varepsilon_{xs}\eta_{xs}] \} \\ &= E_{xs} \{ \mu_{xs}^2 + \mu_{xs}E_{F_{xs}}[\varepsilon_{xs}] + \mu_{xs}E_{G_{xs}}[\eta_{xs}] + E_{F_{xs}G_{xs}}[\varepsilon_{xs}\eta_{xs}] \} \\ &= E_{xs}[\mu_{xs}^2 + \mu_{xs}\Delta\mu_{xs}], \end{aligned}$$

where we used $E_{F_{xs}}[\varepsilon_{xs}] = 0$ and $E_{F_{xs}G_{xs}}[\varepsilon_{xs}\eta_{xs}] = E_{G_{xs}}[E_{F_{xs}}[\varepsilon_{xs}\eta_{xs}]] = E_{G_{xs}}[\eta_{xs}E_{F_{xs}}[\varepsilon_{xs}]] = 0$, due to the independence of w_{xs}^{post} and w_{xs}^{pre} conditional on type t and state s .

The second term on the RHS in Equation 10 becomes

$$\begin{aligned} E[w^{post}]E[w^{pre}] &= E_{xs} \{ E_{F_{xs}}[w_{xs}^{post}] \} E_{xs} \{ E_{G_{xs}}[w_{xs}^{pre}] \} \\ &= E_{xs}[\mu_{xs}]E_{xs}[\mu_{xs} + \Delta\mu_{xs}]. \end{aligned}$$

We finish the derivation by combining the two terms and putting them back into Equation 10

$$\begin{aligned}
Cov(w^{post}, w^{pre}) &= E_{xs}[\mu_{xs}^2] + E_{xs}[\mu_{xs}\Delta\mu_{xs}] - E_{xs}[\mu_{xs}]E_{xs}[\mu_{xs}] - E_{xs}[\mu_{xs}]E_{xs}[\Delta\mu_{xs}] \\
&= E_{xs}[\mu_{xs}^2] - E_{xs}[\mu_{xs}]E_{xs}[\mu_{xs}] + E_{xs}[\mu_{xs}\Delta\mu_{xs}] - E_{xs}[\mu_{xs}]E_{xs}[\Delta\mu_{xs}] \\
&= Var(\mu) + Cov(\mu, \Delta\mu).
\end{aligned}$$

A.2 The Frictional Wage-Growth Statistic and κ_x

In order to compare our statistic to the literature, we would like to relate the wage-growth statistic to fundamental parameters from on-the-job search models, namely the time-invariant Poisson job separation rate (δ_x) and the time-invariant Poisson on-the-job offer rate (λ_x^e). These two parameters determine the probability distribution of the number of job offers a worker receives during an employment spell. Specifically, the probability of receiving $n - 1$ additional offers is

$$Pr_x(n) = \left(\frac{\lambda_x^e}{\lambda_x^e + \delta_x} \right)^{n-1} \frac{\delta_x}{\lambda_x^e + \delta_x}$$

from the literature on Poisson processes (see *e.g.* [Gallager 2013](#)).⁷¹ Recall that the probability of earning a lower wage after being displaced conditional on receiving n job offers is $Pr(w^{post} < w^{pre}|n) = \frac{n}{n+1}$. The relationship between the wage-growth statistic and the Poisson parameters of the on-the-job search model is then

$$\begin{aligned}
Pr_x(w_x^{post} < w_x^{pre}) &= \frac{\delta_x}{\lambda_x^e + \delta_x} \sum_{n=1}^{\infty} \frac{n}{n+1} \left(\frac{\lambda_x^e}{\lambda_x^e + \delta_x} \right)^{n-1} \\
&= \frac{1}{\kappa_x} \sum_{n=0}^{\infty} \frac{n}{n+1} \left(\frac{\kappa_x}{\kappa_x + 1} \right)^n \\
&= \frac{1}{\kappa_x} \left[\sum_{n=0}^{\infty} \left(\frac{\kappa_x}{\kappa_x + 1} \right)^n - \sum_{n=0}^{\infty} \frac{1}{n+1} \left(\frac{\kappa_x}{\kappa_x + 1} \right)^n \right] \\
&= 1 - \frac{(\kappa_x + 1) \ln(\kappa_x + 1) - \kappa_x}{\kappa_x^2},
\end{aligned}$$

where $\kappa_x = \lambda_x^e/\delta_x$, the geometric series $\sum_{n=0}^{\infty} \frac{1}{r^n} = \frac{1}{1-r}$, and $\ln(z) = \sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{z-1}{z} \right)^n$.

⁷¹The probability that the k^{th} arrival of Poisson process 1 occurs before the j^{th} arrival of Poisson process 2 is

$$Pr(S_k^1 < S_j^2) = \sum_{i=k}^{k+j-1} \binom{k+j-1}{i} \left(\frac{\lambda_1}{\lambda_1 + \lambda_2} \right)^i \left(\frac{\lambda_2}{\lambda_1 + \lambda_2} \right)^{k+j-1-i}.$$

A.3 Extension to Sequential-Auction Model

Proposition 2.1 Consider a job offer history for a worker of type x , where the highest pre-displacement offer is p_1^{pre} , the second-highest pre-displacement offer is at least as large as the flow value of unemployment $p_1^{pre} \geq p_2^{pre} \geq b_x$, and the post-displacement offer is p_1^{post} . If $p_1^{post} < p_1^{pre}$ and $\beta_x > \frac{\lambda_x}{2\lambda_x + \rho_x}$, then $w(p_1^{post}, b_x) < w(p_1^{pre}, p_2^{pre})$.

Proof. The proof proceeds in two steps. First, we show that if $\beta_x > \frac{\lambda_x}{2\lambda_x + \rho_x}$, then $\frac{\partial w}{\partial p_1} > 0$. Second, we show that if $\frac{\partial w}{\partial p_1} > 0$, then the $w(p_1^{post}, b_x) < w(p_1^{pre}, p_2^{pre})$ if $p_1^{post} < p_1^{pre}$.

First, the derivative of Equation 8 with respect to the highest wage received, p_1 , is

$$\frac{\partial w}{\partial p_1} = 1 - (1 - \beta_x) \frac{\rho_x + \lambda_x \bar{F}_x(p_1)}{\rho_x + \beta_x \lambda_x \bar{F}_x(p_1)}$$

We want to find a value of β_x such that the derivative is positive for all values of p_1 . It is easy to see that the derivative is positive when $\beta_x = 1$ or when $p_1 = p_x^{max}$ ($\bar{F}_x(p_x^{max}) = 0$). Also note that the derivative is increasing in both p_1 and β_x . Hence, we look for a lower bound on β_x such that the derivative is positive when $p_1 = p_x^{min}$ ($\bar{F}_x(p_x^{min}) = 1$).

$$\begin{aligned} \frac{\partial w}{\partial p_1} \geq \frac{\partial w}{\partial p_1}(p_x^{min}) &= 1 - (1 - \beta_x) \frac{\rho_x + \lambda_x}{\rho_x + \beta_x \lambda_x} > 0 \\ \rho_x + \lambda_x \beta_x - (1 - \beta_x)(\rho_x + \lambda_x) &> 0 \\ \beta_x &> \frac{\lambda_x}{2\lambda_x + \rho_x}. \end{aligned}$$

In the second step, we show that $w(p_1^{post}, b_x) \leq w(p_1^{pre}, p_2^{pre})$ for $p_1^{post} = p_1^{pre}$. Let $\Phi_x(z) = \frac{\rho_x + \lambda_x \bar{F}_x(z)}{\rho_x + \lambda_x \beta_x \bar{F}_x(z)}$. We can write the wage $w(p_1^{pre}, b_x)$ in terms of $w(p_1^{pre}, p_2^{pre})$

$$\begin{aligned} w(p_1^{pre}, b_x) &= p_1^{pre} - (1 - \beta_x) \int_{b_x}^{p_1^{pre}} \Phi_x(z) dz \\ &= p_1^{pre} - (1 - \beta_x) \int_{p_2^{pre}}^{p_1^{pre}} \Phi_x(z) dz - (1 - \beta_x) \int_{b_x}^{p_2^{pre}} \Phi_x(z) dz \\ w(p_1^{pre}, b_x) &= w(p_1^{pre}, p_2^{pre}) - (1 - \beta_x) \int_{b_x}^{p_2^{pre}} \Phi_x(z) dz. \end{aligned}$$

Since $(1 - \beta_x) \int_{b_x}^{p_2^{pre}} \Phi_x(z) dz \geq 0$, we have $w(p_1^{pre}, b_x) \leq w(p_1^{pre}, p_2^{pre})$. Now consider the case when $\beta_x > \frac{\lambda_x}{2\lambda_x + \rho_x}$ and then $\frac{\partial w}{\partial p_1} > 0$. It follows that $w(p_1^{post}, b_x) < w(p_1^{pre}, p_2^{pre}) \quad \forall p_1^{post} < p_1^{pre}$. \square